MATHEMATICS (PART–II)

BOARD'S QUESTION PAPER (MARCH 2024)

Time : 2 Hours] **[Total Marks : 40**

Note : (i) **All** questions are compulsory.
(ii) Use of a calculator is **not** allowed.
(iii) The numbers to the right of the questions indicate full marks.
(iv) In case of MCQs [Q. No. **1(A)**], only the first attempt will be evaluated and will be given credit.
(v) Draw the proper figures wherever necessary.
(vi) The marks of construction should be clear. Do not erase them.
(vii) Diagram is essential for writing the proof of the theorem.

Q. 1. (A) Four alternative answers for each of the following subquestions are given. Choose the correct alternative and write its alphabet : 4

(i) Out of the dates given below which date constitutes a Pythagorean triplet?
(A) 15/8/17 (B) 16/8/16 (C) 3/5/17 (D) 4/9/15

(ii) $\sin\theta \times \operatorname{cosec}\theta = ?$
(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) $\sqrt{2}$

(iii) Slope of X-axis is ....
(A) 1 (B) −1 (C) 0 (D) Cannot be determined

(iv) A circle having radius 3 cm, then the length of its largest chord is ....
(A) 1.5 cm (B) 3 cm (C) 6 cm (D) 9 cm

Q. 1. (B) Solve the following subquestions :

(i) If $\triangle ABC \sim \triangle PQR$ and $AB : PQ = 2 : 3$, then find the value of $\frac{A(\triangle ABC)}{A(\triangle PQR)}$.

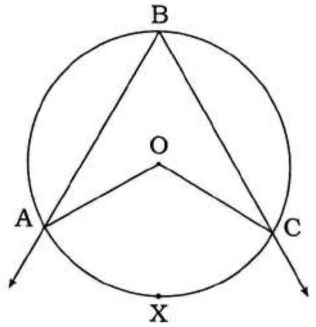
(ii) Two circles of radii 5 cm and 3 cm touch each other externally. Find the distance between their centres.

(iii) Find the side of a square whose diagonal is $10\sqrt{2}$ cm.

(iv) Angle made by the line with the positive direction of X-axis is 45°. Find the slope of that line.

Q. 2. (A) Complete *any two* activities and rewrite it :

(i)



In the above figure, $\angle ABC$ is inscribed in arc ABC. If $\angle ABC = 60°$, find $m\angle AOC$.

(iii) Find the side of a square whose diagonal is $10\sqrt{2}$ cm.

Ans.

Diagonal of square $= \sqrt{2} \times \text{side}$

$10\sqrt{2} = \sqrt{2} \times \text{side}$

$\frac{10\sqrt{2}}{\sqrt{2}} = \text{side}$

$\therefore \text{side} = 10$

The side of the square is **10 cm**.

(iv) Angle made by the line with the positive direction of X-axis is 45°. Find the slope of that line.

Ans.

Angle made by a line with the positive direction of X-axis $(\theta) = 45°$.

Slope of the line $= \tan\theta$

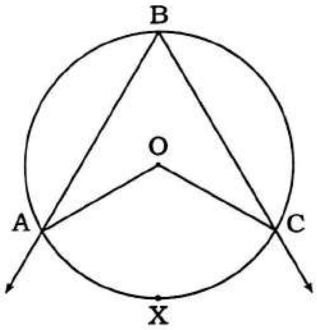
$= \tan 45°$

$= 1.$

The slope of the line is **1**.

Q. 2. (A) Complete *any two* activities and rewrite it :

(i)



In the above figure, $\angle ABC$ is inscribed in arc ABC. If $\angle ABC = 60°$, find $m\angle AOC$.

Activity :

$\angle ABC = \frac{1}{2} m(\text{arc AXC})$ ... **(Inscribed angle theorem)**

$\therefore 60° = \frac{1}{2} m(\text{arc AXC})$

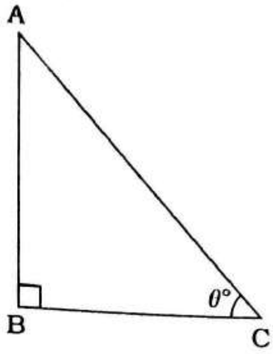
120° $= m(\text{arc AXC})$

But $m\angle AOC =$ m(arc **AXC**) ... (Property of central angle)

$\therefore m\angle AOC =$ **120°**

[**Note :** *From here onwards, answers for activities are provided in bold. Students should consider rewriting the entire activity, including answers, during exams.*]

(ii) Find the value of $\sin^2\theta + \cos^2\theta$.



Activity :

In $\triangle ABC$, $\angle ABC = 90°$, $\angle C = \theta°$.

$AB^2 + BC^2 = \boxed{AC^2}$ ... (Pythagoras theorem)

Dividing both the sides by AC^2,

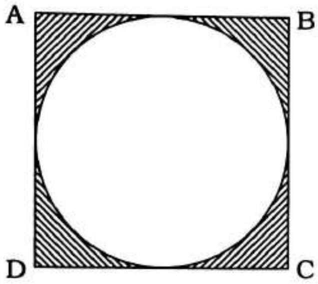
$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

$$\therefore \left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = 1$$

But $\frac{AB}{AC} = \boxed{\sin\theta}$ and $\frac{BC}{AC} = \boxed{\cos\theta}$

$\therefore \sin^2\theta + \cos^2\theta = \boxed{1}$

(iii)



In the figure given above, □ABCD is a square and a circle is inscribed in it. All sides of a square touch the circle.

If AB = 14 cm, find the area of shaded region.

Activity :

AB = 14 cm

Area of square $= \left(\boxed{\text{side}}\right)^2$ ... (Formula)

$= 14^2$

$= \boxed{196}$ cm²

Area of circle $= \boxed{\pi r^2}$ ... (Formula)

$= \frac{22}{7} \times 7 \times 7 = 154$ cm²

(Area of shaded portion) = (Area of square) − (Area of circle) ... (Formula)

$= 196 - 154$

$= \boxed{42}$ cm²

Q. 2. (B) Solve *any four* of the following subquestions :

(i) Radius of a sector of a circle is 3.5 cm and length of its arc is 2.2 cm. Find the area of the sector.

Solution :

Radius of circle $(r) = 3.5$ cm

Length of arc $(l) = 2.2$ cm

$$\text{Area of sector} = \frac{1}{2} \times l \times r$$

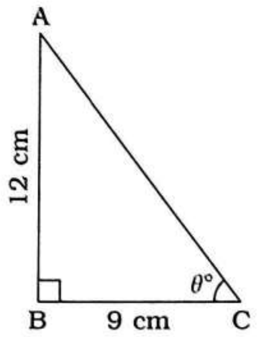
$$= \frac{1}{2} \times 2.2 \times 3.5$$

$$= 3.85 \text{ cm}^2$$

Ans. Area of sector is **3.85 cm²**.

(ii) Find the length of the hypotenuse of a right-angled triangle, if remaining sides are 9 cm and 12 cm.

Solution :



Let $\triangle$ABC be the given right-angled triangle. AB = 12 cm and BC = 9 cm.

In $\triangle$ABC,

$\angle ABC = 90°$

$\therefore$ by Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\therefore AC^2 = 12^2 + 9^2$

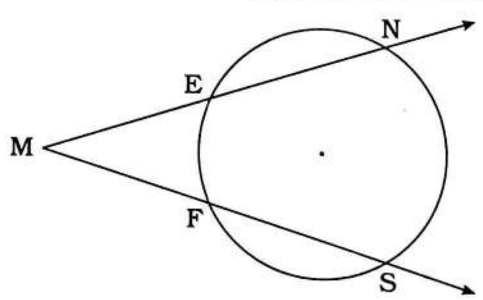
$\therefore AC^2 = 144 + 81$

$\therefore AC^2 = 225$

$\therefore AC = 15$ cm ... (Taking square roots of both the sides)

Ans. Hypotenuse of right-angled triangle is **15 cm**.

(iii)



In the above figure, $m(\text{arc NS}) = 125°$, $m(\text{arc EF}) = 37°$. Find the measure of $\angle NMS$.

Solution :

The vertex of $\angle NMS$ is in exterior of the given circle and it intercepts arc NS and arc EF.

$m\angle NMS = \frac{1}{2}[m(\text{arc NS}) - m(\text{arc EF})]$

$= \frac{1}{2}[125° - 37°]$

$= \frac{1}{2} \times 88°$

$= 44°$

Ans. $m\angle NMS =$ **44°**.

(iv) Find the slope of the line passing through the points A(2, 3), B(4, 7).

Solution :

$A(2, 3) \equiv (x_1, y_1)$

$B(4, 7) \equiv (x_2, y_2)$

Slope of AB $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7-3}{4-2}$

$= \frac{4}{2}$

$= 2$

Ans. Slope of line AB is **2**.

(v) Find the surface area of a sphere of radius 7 cm.

Solution :

Radius of sphere $(r) = 7$ cm

Surface area of sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 7^2$

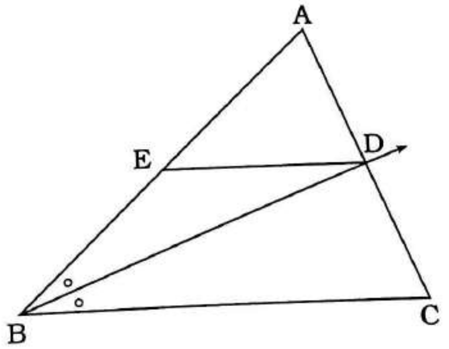
$= 4 \times 22 \times 7$

$= 616 \text{ cm}^2$

Ans. Surface area of sphere is **616 cm²**.

Q. 3. (A) Complete *any one* activity of the following and rewrite it :

(i)



In $\triangle ABC$, ray BD bisects $\angle ABC$, A–D–C, seg DE ∥ side BC, A–E–B, then for showing $\frac{AB}{BC} = \frac{AE}{EB}$, complete the following activity :

Proof :

In $\triangle ABC$, ray BD bisects $\angle B$.

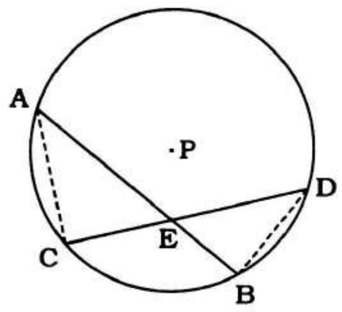
$\therefore \frac{\boxed{AB}}{BC} = \frac{AD}{DC}$ ... (1) (By theorem of angle bisector of triangle)

In $\triangle ABC$, DE $\parallel$ BC.

$\therefore \frac{\boxed{AE}}{EB} = \frac{AD}{DC}$ ... (2) (By Basic Proportionality theorem)

$\therefore \frac{AB}{\boxed{BC}} = \frac{\boxed{AE}}{EB}$ ... [From (1) and (2)]

(ii)



Given :

Chords AB and CD of a circle with centre P intersect at point E.

To prove : $AE \times EB = CE \times ED$

Construction :

Draw seg AC and seg BD.

Fill in the blanks and complete the proof.

Proof :

In $\triangle CAE$ and $\triangle BDE$,

$\angle AEC \cong \angle DEB$ ... **Vertically opposite angles**

$\boxed{\angle CAE} \cong \angle BDE$ ... (Angles inscribed in the same arc)

$\therefore \triangle CAE \sim \triangle BDE$ ... **AA test of similarity**

$\therefore \frac{\boxed{AE}}{DE} = \frac{CE}{\boxed{BE}}$ ... **Corresponding sides of similar triangles**

$\therefore AE \times EB = CE \times ED$.

Q. 3. (B) Solve *any two* of the following subquestions :

(i) Determine whether the points are collinear.

A(1, −3), B(2, −5), C(−4, 7)

Solution :

A(1, −3), B(2, −5), C(−4, 7).

Let $A(1, -3) \equiv (x_1, y_1)$, $B(2, -5) \equiv (x_2, y_2)$ and $C(-4, 7) \equiv (x_3, y_3)$

Slope of line $AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-(-3)}{2-1} = \frac{-5+3}{1} = -2$

Slope of line BC $= \dfrac{y_3 - y_2}{x_3 - x_2} = \dfrac{7-(-5)}{-4-2} = \dfrac{7+5}{-6} = \dfrac{12}{-6} = -2$

$\therefore$ Slope of line AB = slope of line BC and B is the common point.

Ans. Points A, B and C **are collinear.**

(ii) $\triangle ABC \sim \triangle LMN$. In $\triangle ABC$, AB = 5.5 cm, BC = 6 cm, CA = 4.5 cm. Construct $\triangle ABC$ and $\triangle LMN$ such that $\dfrac{BC}{MN} = \dfrac{5}{4}$.

Solution :

For $\triangle ABC$, the lengths of three sides are known.

$\therefore \triangle ABC$ can be constructed.

$\triangle ABC \sim \triangle LMN$

$\therefore \dfrac{AB}{LM} = \dfrac{BC}{MN} = \dfrac{AC}{LN}$ ... (Corresponding sides of similar triangles are in proportion)

$\therefore \dfrac{5.5}{LM} = \dfrac{6}{MN} = \dfrac{4.5}{LN} = \dfrac{5}{4}$

$\therefore \dfrac{5.5}{LM} = \dfrac{5}{4}$

$\therefore LM = \dfrac{5.5 \times 4}{5}$

$\therefore LM = 1.1 \times 4$

$\therefore LM = 4.4$ cm

$\dfrac{6}{MN} = \dfrac{5}{4}$

$\therefore MN = \dfrac{6 \times 4}{5}$

$\therefore MN = \dfrac{24}{5}$

$\therefore MN = 4.8$ cm

$\dfrac{4.5}{LN} = \dfrac{5}{4}$

$\therefore LN = \dfrac{4.5 \times 4}{5}$

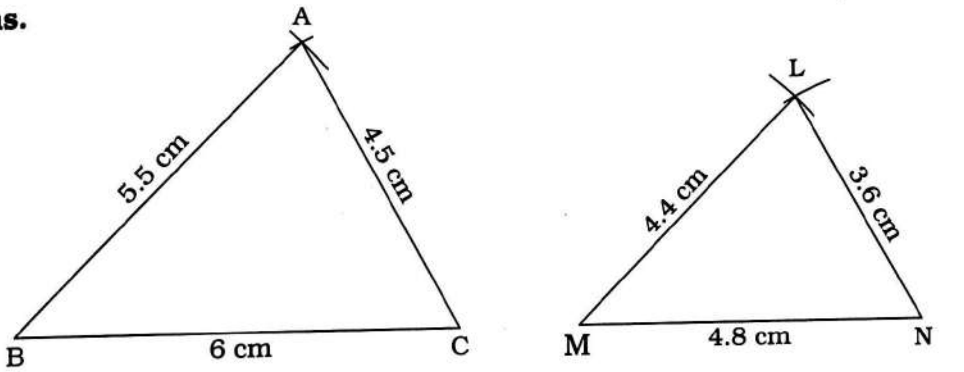
$\therefore LN = \dfrac{18}{5}$

$\therefore LN = 3.6$ cm

For $\triangle LMN$, the lengths of three sides are known.

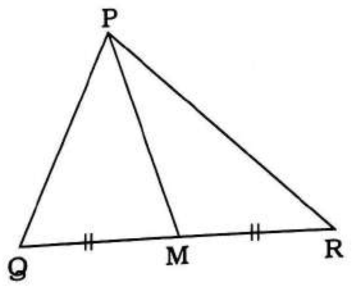
$\therefore \triangle LMN$ can be constructed.

Ans.



(iii) Seg PM is a median of $\triangle PQR$, PM = 9 and $PQ^2 + PR^2 = 290$, then find QR.

Solution :



In $\triangle PQR$,

seg PM is the median.

$PQ^2 + PR^2 = 2PM^2 + 2QM^2$ ... (Apollonius theorem)

$\therefore 290 = 2 \times 9^2 + 2 \times QM^2$

$\therefore 290 = 162 + 2QM^2$

$\therefore 2QM^2 = 290 - 162$

$\therefore 2QM^2 = 128$

$\therefore QM^2 = 64$

$\therefore QM = \sqrt{64}$

$\therefore QM = 8$

$QR = 2 \times QM$ ... [M is the midpoint of QR]

$\therefore QR = 2 \times 8$

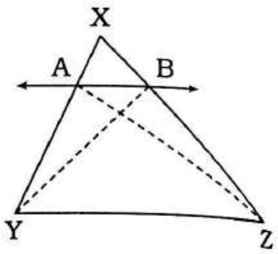
$\therefore QR = 16$ units

Ans. QR = **16 units**

(iv) Prove that, 'If a line parallel to a side of a triangle intersects the remaining sides in two distinct points, then the line divides the side in the same proportion'.

Solution :

Statement : If a line parallel to a side of a triangle intersects the remaining sides in two distinct points, then the line divides the sides in the same proportion.



Given : In $\triangle XYZ$,

(i) Line AB $\parallel$ side YZ.

(ii) Line AB intersects side XY and side XZ in points A and B respectively such that X–A–Y and X–B–Z.

To prove : $\frac{XA}{AY} = \frac{XB}{BZ}$.

Construction : Draw seg BY and seg AZ.

Proof : $\triangle XAB$ and $\triangle BAY$ have a common vertex B and their bases XA and AY lie on the same line XY.

$\therefore$ they have equal heights.

$\therefore \frac{A(\triangle XAB)}{A(\triangle BAY)} = \frac{XA}{AY}$ ... (Triangles of equal heights) ... (1)

$\triangle XAB$ and $\triangle ABZ$ have a common vertex A and their bases XB and BZ lie on the same line XZ.

$\therefore$ they have equal heights.

$\therefore \frac{A(\triangle XAB)}{A(\triangle ABZ)} = \frac{XB}{BZ}$ ... (Triangles of equal heights) ... (2)

$\triangle BAY$ and $\triangle ABZ$ lie between the same two parallel lines AB and YZ.

$\therefore$ they have equal heights, also they have same base AB.

$\therefore A(\triangle BAY) = A(\triangle ABZ)$ ... [Triangles with same base and equal heights] ... (3)

$\therefore$ from (1), (2) and (3), we get

$\frac{A(\triangle XAB)}{A(\triangle BAY)} = \frac{A(\triangle XAB)}{A(\triangle ABZ)}$ ... (4)

$\therefore$ from (1), (2) and (4), we get

$\frac{XA}{AY} = \frac{XB}{BZ}$.

Q. 4. **Solve any two of the following subquestions :**

(i) $\frac{1}{\sin^2\theta}-\frac{1}{\cos^2\theta}-\frac{1}{\tan^2\theta}-\frac{1}{\cot^2\theta}-\frac{1}{\sec^2\theta}-\frac{1}{\operatorname{cosec}^2\theta}=-3$, then find the value of θ.

Solution :

$$\frac{1}{\sin^2\theta}-\frac{1}{\cos^2\theta}-\frac{1}{\tan^2\theta}-\frac{1}{\cot^2\theta}-\frac{1}{\sec^2\theta}-\frac{1}{\operatorname{cosec}^2\theta}=-3$$

$$\therefore \operatorname{cosec}^2\theta-\sec^2\theta-\cot^2\theta-\tan^2\theta-\cos^2\theta-\sin^2\theta=-3$$

... $[\because \frac{1}{\sin\theta}=\operatorname{cosec}\theta,\ \frac{1}{\cos\theta}=\sec\theta,\ \frac{1}{\tan\theta}=\cot\theta,\ \frac{1}{\cot\theta}=\tan\theta,\ \frac{1}{\sec\theta}=\cos\theta$

and $\frac{1}{\operatorname{cosec}\theta}=\sin\theta]$

$$\therefore (\operatorname{cosec}^2\theta-\cot^2\theta)-(\cos^2\theta+\sin^2\theta)-(\sec^2\theta+\tan^2\theta)=-3$$

$$\therefore 1-1-(\sec^2\theta+\tan^2\theta)=-3$$

$$\because \begin{bmatrix}\operatorname{cosec}^2\theta=1+\cot^2\theta,\ \operatorname{cosec}^2\theta-\cot^2\theta=1\\ \sin^2\theta+\cos^2\theta=1\end{bmatrix}$$

$$\therefore (\sec^2\theta+\tan^2\theta)=3$$

$$\therefore 1+\tan^2\theta+\tan^2\theta=3 \qquad \text{... } [\sec^2\theta=1+\tan^2\theta]$$

$$\therefore 2\tan^2\theta=3-1$$

$$\therefore 2\tan^2\theta=2$$

$$\therefore \tan^2\theta=1$$

$$\therefore \tan\theta=1$$

we know $\tan 45°=1$

$$\therefore \theta=45°$$

Ans. $\theta=\mathbf{45°}$

(ii) A cylinder of radius 12 cm contains water up to the height 20 cm. A spherical iron ball is dropped into the cylinder and thus water level raised by 6.75 cm. What is the radius of iron ball?

Solution :

Let the radius of the spherical ball be (R).

Radius of the cylinder $(r)=12$ cm. Height of the raised water $(h)=6.75$ cm.

Volume of iron ball = Volume of water raised

$$\therefore \frac{4}{3}\pi R^3=\pi r^2 h$$

$$\therefore \frac{4}{3}R^3=r^2 h$$

$$\therefore \frac{4}{3}\times R^3=12\times 12\times 6.75$$

$$\therefore R^3=\frac{12\times 12\times 6.75\times 3}{4}$$

$$\therefore R^3=729$$

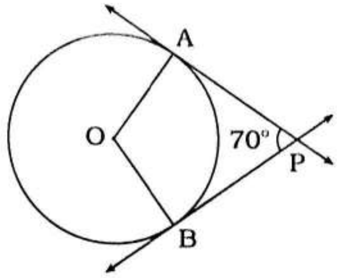
$$\therefore R=9 \text{ cm}$$

Ans. Radius of the iron ball is **9 cm.**

(iii) Draw a circle with centre O having radius 3 cm. Draw tangent segments PA and PB through the point P outside the circle such that $\angle APB = 70^\circ$.

Solution :

Rough figure :



Analytical figure :

$\angle OAP = \angle OBP = 90^\circ$ ... (Tangent theorem)

$\angle OAP + \angle OBP + \angle APB + \angle AOB = 360^\circ$

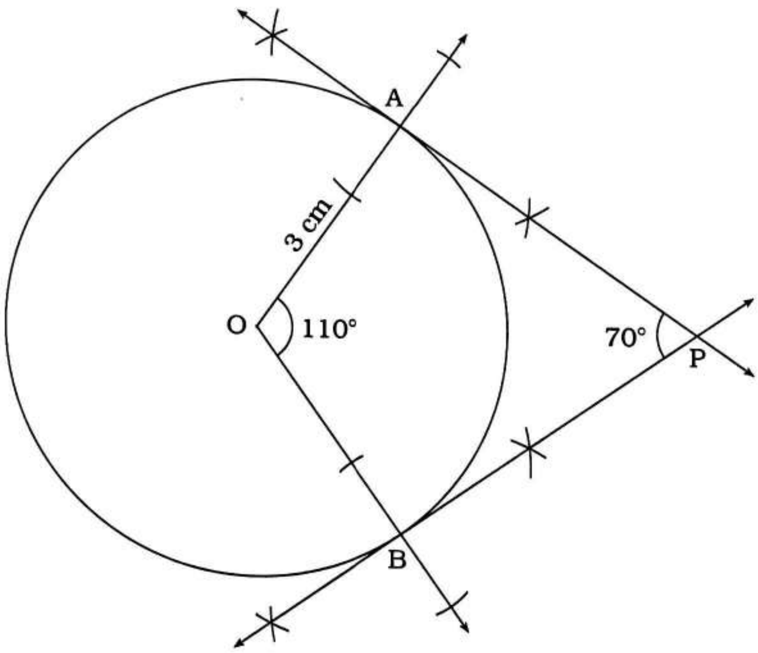
... (Sum of all angles of a quadrilateral is 360°)

$90^\circ + 90^\circ + 70^\circ + \angle AOB = 360^\circ$

$\therefore \angle AOB = 360^\circ - 250^\circ$

$\therefore \angle AOB = 110^\circ$

Ans.



Q. 5. Solve *any one* of the following subquestions :

(i) □ABCD is trapezium, AB ∥ CD diagonals of trapezium intersects in point P. Write the answers of the following questions :

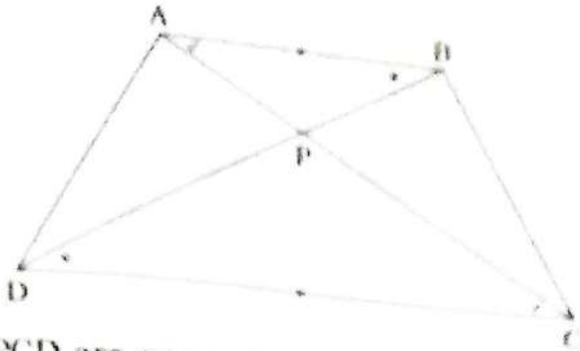
(a) Draw the figure using the given information.

(b) Write any one pair of alternate angles and opposite angles.

(c) Write the names of similar triangles with test of similarity.

Solution :

(a)



(b) ∠PAB and ∠PCD are one pair of alternate angles

(Note : Student can even write ∠PBA and ∠PDC)

∠APB and ∠CPD are one pair of opposite angles

(Note : Student can even write ∠BPC and ∠APD)

(c) $\triangle APB \sim \triangle CPD$ ... (AA test of similarity)

(ii) AB is a chord of a circle with centre O. AOC is diameter of circle. AT is a tangent at A.

Write answers of the following questions :

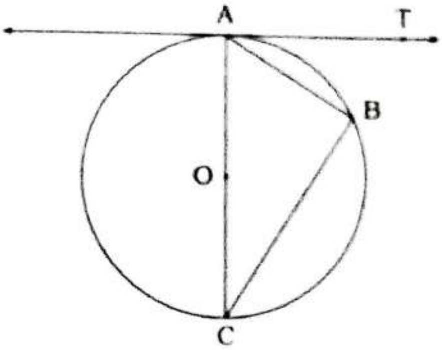
(a) Draw the figure using given information.

(b) Find the measures of ∠CAT and ∠ABC with reasons.

(c) Whether ∠CAT and ∠ABC are congruent? Justify your answer.

Solution :

(a)



(b) $\angle CAT = 90^\circ$ ... (Tangent theorem)

$\angle ABC = 90^\circ$ ... (Angle inscribed in a semicircle is a right angle)

(c) $\angle CAT \cong \angle ABC$ ... (Both measure 90°)

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