SOLUTION

Q.1. (A) Four alternative answers are given for every sub-question. Select the correct alternative and write the alphabet of that answer: [4]

(1) If a, b, c are sides of a triangle and $a^2 + b^2 = c^2$, name the type of triangle: [1]

(a) Obtuse angled triangle (b) Acute angled triangle
(c) Right angled triangle (d) Equilateral triangle

(2) Chords AB and CD of a circle intersect inside the circle at point E. If AE = 4, EB = 10, CE = 8, then find ED: [1]

(a) 7 (b) 5
(c) 8 (d) 9

(3) Co-ordinates of origin are .... [1]

(a) (0, 0) (b) (0, 1) (c) (1, 0) (d) (1, 1)

(4) If radius of the base of cone is 7 cm and height is 24 cm, then find its slant height: [1]

(a) 23 cm (b) 26 cm (c) 31 cm (d) 25 cm

Ans. (1) – (c), (2) – (b), (3) – (a), (4) – (d)

Q.1. (B) Solve the following sub-questions. **[4]**

(1) If $\Delta ABC \sim \Delta PQR$ and $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{16}{25}$, then find AB:PQ.

Solution:

$\Delta ABC \sim \Delta PQR$ ...(Given)

$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$ ...(Ratio of the areas of two similar triangles) [½]

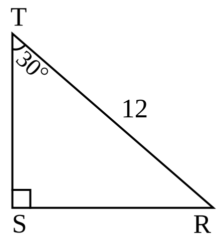
$\therefore \frac{AB^2}{PQ^2} = \frac{16}{25}$

$\therefore \frac{AB}{PQ} = \frac{4}{5}$ (Taking square roots) [½] [1]

Ans. $\therefore$ AB:PQ = 4:5

(2) In ΔRST, $\angle S = 90°$, $\angle T = 30°$, RT = 12 cm, then find RS.

Solution:



In ΔRST,

$\angle S = 90°$ and $\angle T = 30°$ ...(Given)

$\therefore \angle R = 60°$ ...(Remaining angle)

$\therefore$ By 30°– 60°– 90° theorem,

$RS = \frac{1}{2} RT$ ...(Side opposite 30°) [½]

$\therefore RS = \frac{1}{2} \times 12$

Ans. $\therefore$ RS = 6 cm [½] [1]

(3) If radius of a circle is 5 cm, then find the length of the longest chord of the circle.

Solution:

Radius = 5 cm

We know that the longest chord of a circle is a diameter. [½]

diameter $= 2 \times$ radius

$\therefore$ diameter $= 2 \times 5$

$= 10$ cm [½][1]

Ans. $\therefore$ The length of the longest chord is 10 cm.

(4) Find the distance between the points O(0, 0) and P(3, 4).

Solution:

$O(0, 0) \equiv (x_1, y_1)$

$P(3, 4) \equiv (x_2, y_2)$

By distance formula,

$$d(O, P) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ [½]

$$= \sqrt{(3-0)^2 + (4-0)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$\therefore \quad d(O, P) = 5$ [½] [1]

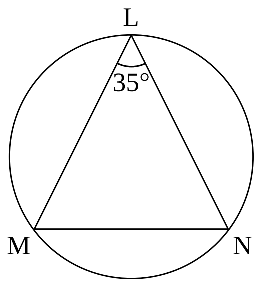
Ans. $\therefore$ The distance between the two given points is 5 units.

Q.2. (A) Complete the following activities. (Any *two*) **[4]**

(1) In the given figure, $\angle L = 35°$,

find:

a. m(arc MN) b. m(arc MLN)



Solution:

a. $\angle L = \frac{1}{2} m(\text{arc MN})$

...(By inscribed angle theorem)

$\therefore$ [35°] $= \frac{1}{2} m(\text{arc MN})$ [½]

$\therefore \quad 2 \times 35 = m(\text{arc MN})$

$\therefore \quad m(\text{arc MN}) =$ [70°] [½]

b. m(arc MLN) = [360°] – m(arc MN) [½]

...[Definition of measure of an arc]

$= 360° - 70°$

Ans. $\therefore$ m(arc MLN) = [290°] [½] [2]

(2) Show that $\cot\theta + \tan\theta = \operatorname{cosec}\theta \times \sec\theta$

Solution:

$$\text{L.H.S} = \cot\theta + \tan\theta$$

$$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\boxed{\cos^2\theta} + \boxed{\sin^2\theta}}{\sin\theta \times \cos\theta}$$ [½ + ½]

$$= \frac{1}{\sin\theta \times \cos\theta} \quad \cdots \boxed{\sin^2\theta + \cos^2\theta = 1}$$ [½]

$$= \frac{1}{\sin\theta} \times \frac{1}{\boxed{\cos\theta}}$$ [½]

$$= \operatorname{cosec}\theta \times \sec\theta$$

L.H.S.= R.H.S. [2]

$\therefore \cot\theta + \tan\theta = \operatorname{cosec}\theta \times \sec\theta.$

(3) Find the surface area of a sphere of radius 7 cm.

Solution:

Surface area of sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times \boxed{7}^2$$ [½]

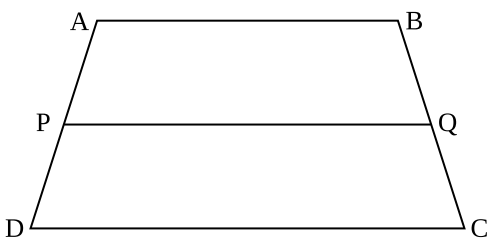
$$= 4 \times \frac{22}{7} \times \boxed{49}$$ [½]

$$= \boxed{88} \times 7$$ [½]

Ans. $\therefore$ Surface area of sphere = $\boxed{616}$ sq.cm. [½] [2]

Q.2. (B) Solve the following sub-questions. (Any *four*) [8]

(1)



In trapezium ABCD side AB || side PQ || side DC. AP = 15, PD = 12, QC = 14, find BQ.

Solution:

$AB \parallel PQ \parallel DC$ (Given)

$\therefore \frac{AP}{PD} = \frac{BQ}{QC}$ (Intercepts made by three parallel lines) [½]

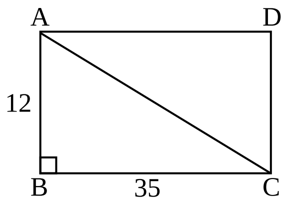
$\therefore \frac{15}{12} = \frac{BQ}{14}$ [½]

$\therefore BQ = \frac{15 \times 14}{12}$ [½]

Ans. $\therefore BQ = 17.5$ [½] [2]

(2) Find the length of the diagonal of a rectangle whose length is 35 cm and breadth is 12 cm.

Solution:



Let □ABCD be the rectangle.

$\angle ABC = 90°$ ...(Angle of a rectangle)

$\therefore$ In ΔABC, $\angle ABC = 90°$

$\therefore AC^2 = AB^2 + BC^2$

...(Pythagoras theorem) [½]

$\therefore AC^2 = 12^2 + 35^2$

$\therefore AC^2 = 144 + 1225$ [½]

$\therefore AC^2 = 1369$ [½]

$\therefore AC = 37$ ...(Taking square roots) [½] [2]

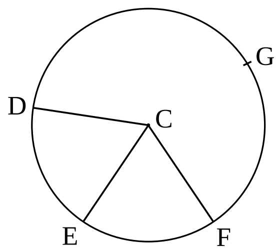
Ans. The length of the diagonal is 37 cm.

(3) In the given figure, points G, D, E, F are points of a circle with centre C, $\angle ECF = 70°$, $m(\text{arc } DGF) = 200°$.

Find:

a. $m(\text{arc } DE)$

b. $m(\text{arc } DEF)$



Solution:

a. $\angle DCF = m(\text{arc } DGF) = 200°$ ...(Central angle)

$\angle DCF + \angle DCE + \angle ECF = 360°$ ...(Total angular measure of a circle)

$\therefore 200° + \angle DCE + 70° = 360°$ [½]

$\therefore \quad \angle DCE = 360° – 270°$

$\therefore \quad \angle DCE = 90°$

$m(\text{arc DE}) = m\angle DCE$ ...(Central angle)

Ans. $\therefore\ m(\text{arc DE}) = 90°$ [½]

b. $m(\text{arc EF}) = m\angle ECF$ ...(Central angle)

$\therefore\ m(\text{arc EF}) = 70°$

$m(\text{arc DEF}) = m(\text{arc DE}) + m(\text{arc EF})$

$\therefore\ m(\text{arc DEF}) = 90° + 70°$ [½]

Ans. $\therefore\ m(\text{arc DEF}) = 160°$ [½] [2]

(4) Show that points A(–1, –1), B(0, 1), C(1, 3) are collinear.

Solution:

Let $A(-1, -1) \equiv (x_1, y_1)$

$B(0, 1) \equiv (x_2, y_2)$

Slope of AB $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{1 - (-1)}{0 - (-1)}$

$= \dfrac{2}{1}$

$= \mathbf{2}$ [½]

Let $B(0, 1) \equiv (x_1, y_1)$

$C(1, 3) \equiv (x_2, y_2)$

Slope of BC $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{3 - 1}{1 - 0}$

$= \dfrac{2}{1}$

$= \mathbf{2}$ [½]

Let $A(-1, -1) \equiv (x_1, y_1)$

$C(1, 3) \equiv (x_2, y_2)$

Slope of AC $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{3-(-1)}{1-(-1)}$$

$$= \frac{4}{2}$$

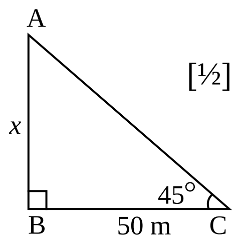
$$= \mathbf{2}$$ [½]

Since the slopes of AB, BC and AC are equal, points A(–1, –1), B(0, 1) and C(1, 3) are collinear.

Hence proved. [½] [2]

(5) A person is standing at a distance of 50 m from a temple looking at its top. The angle of elevation is of 45°. Find the height of the temple.

Solution:



[½]

Let AB be the height of the temple and the person is standing at point 'C'. BC is the distance between the person and the temple.

Angle of elevation = ∠ACB [½]

In ΔABC,

$\angle B = 90°$ ....(The temple is pendicular to the ground)

$\therefore \tan C = \frac{AB}{BC}$

$\therefore \tan 45° = \frac{x}{50}$ ...($\because \angle C = 45°$) [½]

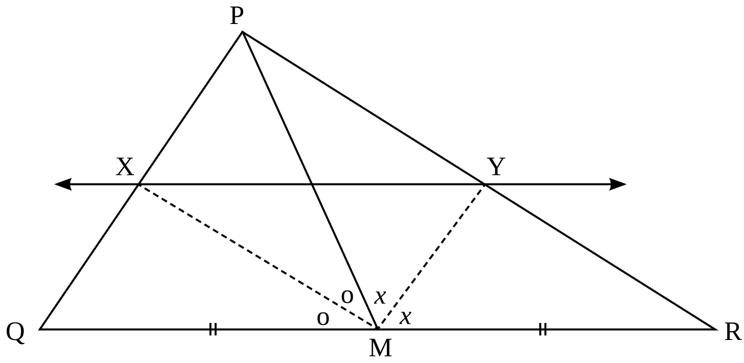
$\therefore \quad 1 = \frac{x}{50}$

$\therefore \quad x = 50$ m [½] [2]

Ans. ∴ The height of the temple is 50 m.

Q.3. (A) Complete the following activities. (Any *one*) **[3]**

(1)



In ΔPQR, seg PM is a median. Angle bisectors of $\angle$PMQ and $\angle$PMR intersect side PQ and side PR in points X and Y respectively. Prove that XY || QR.
Complete the proof by filling in the boxes.

Solution:

In ΔPMQ,
Ray MX is the bisector of $\angle$PMQ.

$\therefore \quad \frac{MP}{MQ} = \frac{\boxed{PX}}{\boxed{QX}}$ ...(I) (Theorem of angle bisector) [½ + ½]

Similarly, in ΔPMR, Ray MY is the bisector of $\angle$PMR.

$\therefore \quad \frac{MP}{MR} = \frac{\boxed{PY}}{\boxed{RY}}$ ...(II) (Theorem of angle bisector) [½ + ½]

But $\frac{MP}{MQ} = \frac{MP}{MR}$ ...(III) (As M is the midpoint of QR)

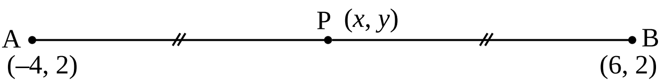
Hence MQ = MR [½]

$\therefore \quad \frac{PX}{\boxed{QX}} = \frac{\boxed{PY}}{YR}$ ...[From (I), (II) and (III)] [½] [3]

$\therefore$ XY || QR ...[Converse of basic proportionality theorem]

(2) Find the co-ordinates of point P where P is the midpoint of a line segment AB with A(–4, 2) and B(6, 2).

Solution:



Suppose, (–4, 2) = (x_1, y_1) and (6, 2) = (x_2, y_2) and co-ordinates of P are (x, y).

$\therefore$ According to midpoint theorem,

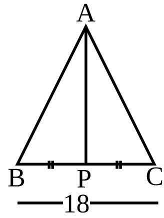
$$x = \frac{x_1 + x_2}{2} = \frac{\boxed{-4} + 6}{2} = \frac{\boxed{2}}{2} = \boxed{1}$$ [½ + ½ + ½]

$$y = \frac{y_1 + y_2}{2} = \frac{2 + \boxed{2}}{2} = \frac{4}{2} = \boxed{2}$$ [½ + ½]

$\therefore$ Co-ordinates of midpoint P are $\boxed{(1, 2)}$ [½] [3]

Q.3. (B) Solve the following sub-questions. (Any *two*) [6]

(1) In ΔABC, seg AP is a median. If BC = 18, $AB^2 + AC^2 = 260$, find AP.



Solution:

In ΔABC, AP is a median.

$\therefore$ BP = PC = 9 [½]

$\therefore$ $AB^2 + AC^2 = 2AP^2 + 2BP^2$ ...(Apollonius theorem) [½]

$\therefore$ $260 = 2(AP^2 + 9^2)$ [½]

$\therefore$ $AP^2 + 81 = \frac{260}{2}$ [½]

$\therefore$ $AP^2 = 130 - 81$ [½]

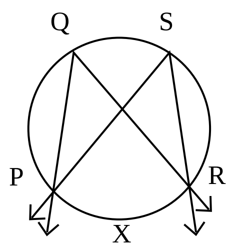
$\therefore$ $AP^2 = 49$

$\therefore$ AP = 7 ...(Taking square roots) [½] [3]

Ans. AP = 7

(2) Prove that "Angles inscribed in the same arc are congruent."

Solution:



Given: $\angle PQR$ and $\angle PSR$ are inscribed in the same arc PQR and their intercepted are is arc PXR. [½]

To Prove: $\angle PQR \cong \angle PSR$ [½]

Proof: $m\angle PQR = \frac{1}{2} m(\text{arc PXR})$ ...(Inscribed angle) ...(1) [½]

$m\angle PSR = \frac{1}{2} m(\text{arc PXR})$ ...(Inscribed angle) ...(2) [½]

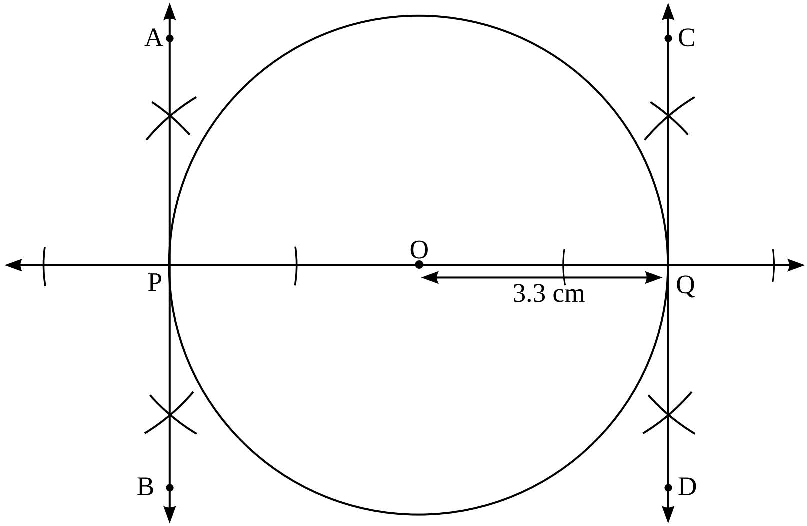
$\therefore$ $m\angle PQR = m\angle PSR$ ...[From (1) and (2)] [½]

$\therefore$ $\angle PQR \cong \angle PSR$ [½] [3]

Hence proved.

(3) Draw a circle of radius 3.3 cm. Draw a chord PQ of length 6.6 cm. Draw tangents to the circle at points P and Q.

Solution:



AB and CD are the tangents at points P and Q respectively.

1. To draw a circle of radius 3.3 cm. [½]
2. To draw a 6.6 cm chord passing through the centre [½]
3. To draw tangents at point P [1]
4. To draw tangents at point Q [1] [3]

(4) The radii of circular ends of a frustum are 14 cm and 6 cm respectively and its height is 6 cm. Find its curved surface area. ($\pi = 3.14$)

Solution:

Here, $r_1 = 14$ cm, $r_2 = 6$ cm and $h = 6$ cm.

Slant height of a frustum (l) $= \sqrt{h^2 + (r_1 - r_2)^2}$ [½]

$= \sqrt{6^2 + (14 - 6)^2}$ [½]

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$\therefore$ $l = 10$ cm [½]

Curved surface area of a frustum $= \pi(r_1 + r_2)l$ [½]

$= 3.14 \times (14 + 6) \times 10$ [½]

$= 3.14 \times 20 \times 10$

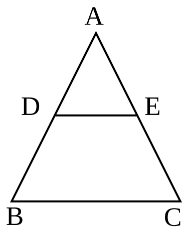
$= 628 \text{ cm}^2$ [½] [3]

Ans. $\therefore$ The curved surface area of the frustum is 628 sq.cm.

Q.4. Solve the following sub-questions. (Any *two*) **[8]**

(1) In ΔABC, seg DE || side BC. If $2A(\Delta ADE) = A(\square DBCE)$, find AB:AD and show that $BC = \sqrt{3}\,DE$.

Solution:



Given: In ΔABC, seg DE || side BC.

To find: AB:AD

To prove: $BC = \sqrt{3}\,DE$

Proof:

$2A(\Delta ADE) = A(\square DBCE)$ ...(Given)

$A(\Delta ABC) = A(\Delta ADE) + A(\square DBCE)$

$\therefore A(\Delta ABC) = A(\Delta ADE) + 2A(\Delta ADE)$ [½]

$= 3A(\Delta ADE)$

$\therefore \dfrac{A(\Delta ABC)}{A(\Delta ADE)} = 3$ ..... (I) [½]

In ΔADE and ΔABC,

$\angle DAE = \angle BAC$ ...(common angles) [½]

$\angle ADE = \angle ABC$ ...(corresponding angles) [½]

$\therefore \Delta ADE \sim \Delta ABC$ ...(By AA test) [½]

$\therefore \dfrac{A(\Delta ABC)}{A(\Delta ADE)} = \dfrac{BC^2}{DE^2}$ (Areas of similar triangles)... (II) [½]

$\therefore \dfrac{BC^2}{DE^2} = 3$ ....[From I and (II)] [½]

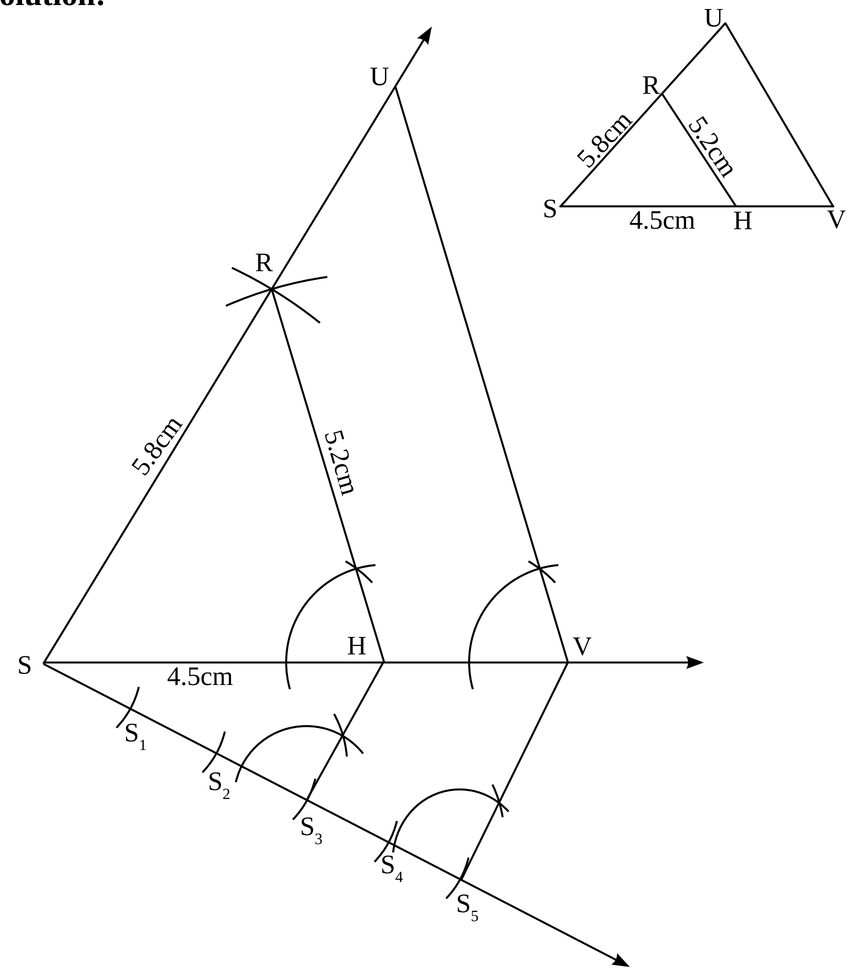
$\therefore \dfrac{BC}{DE} = \sqrt{3}$ ....(Taking square root)

$\therefore BC = \sqrt{3}\,DE$ [½][4]

Hence proved.

(2) $\Delta SHR \sim \Delta SVU$. In ΔSHR, SH = 4.5 cm, HR = 5.2 cm, SR = 5.8 cm and $\dfrac{SH}{SV} = \dfrac{3}{5}$, construct ΔSVU.

Solution:



1.	For rough figure	[1]
2.	For constuction of ΔSHR	[1]
3.	For drawing line $S_5V \| \| S_3H$	[1]
4.	For drawing line YU || HR	[1] [4]

(3) An ice-cream pot has a right circular cylindrical shape. The radius of the base is 12 cm and height is 7 cm. This pot is completely filled with ice-cream. The entire ice-cream is given to the students in the form of right circular ice-cream cones, having diameter 4 cm and height 3.5 cm. If each student is given one cone, how many students can be served?

Solution:

Cylinder: Radius (r_2) = 12 cm and (H) = 7 cm

Cone: Diameter = 4 cm, Radius (r_1) = 2 cm and height (h) = 3.5 cm

Let the number of students be x.

$\therefore$ $x \times$ Volume of a cone = Volume of the cylinder [1]

$\therefore$ $x \times \frac{1}{3}\pi r_1^2 h = \pi r_2^2 H$ [½]

$\therefore$ $\frac{x}{3} \times r_1^2 h = r_2^2 H$ [½]

$\therefore$ $x = \frac{3r_2^2 H}{r_1^2 h}$ [½]

$\therefore$ $x = \frac{3 \times 12 \times 12 \times 7}{2 \times 2 \times 3.5}$ [½]

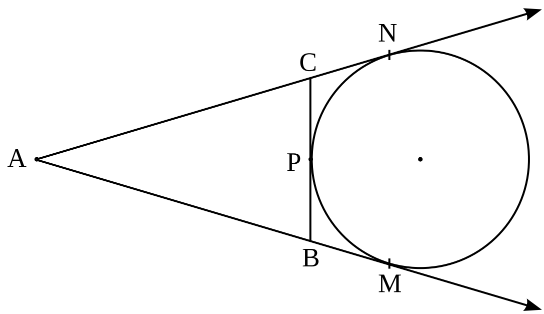
$\therefore$ $x = 12 \times 18$ [½]

$\therefore$ $x = 216$

Ans. $\therefore$ 216 students can be served. [½][4]

Q.5. Solve the following sub-questions. (Any *one*) **[3]**

(1)



A circle touches side BC at point P of ΔABC, from outside of the triangle. Further extended lines AC and AB are tangents to the circle at N and M respectively. Prove that:

$$AM = \frac{1}{2}\text{(Perimeter of } \Delta ABC)$$

Solution:

Perimeter of ΔABC =AB + BC + CA [½]

=AB + (BP + PC) + (AN – CN) [1]

(∵ A–C–N)

Now, BP = BM, AN = AM and CN = PC [½]

...(Tangents drawn from an exterior point are equal)

$\therefore$ Perimeter of $\Delta ABC = (AB + BM) + PC + (AM - PC)$ [½]

$= AM + AM$

$= 2AM$

$\therefore \quad AM = \frac{1}{2}$ (Perimeter of ΔABC) [½][3]

Hence proved.

(2) Eliminate θ if $x = r \cos \theta$ and $y = r \sin \theta$.

Solution:

$x = r \cos \theta$ and $y = r \sin \theta$

$\therefore \quad x^2 = r^2 \cos^2 \theta$ ...(I) [½]

and $\quad y^2 = r^2 \sin^2 \theta$ ...(II) [½]

$\therefore \quad x^2 + y^2 = r^2 \cos^2 \theta + r^2 \sin^2 \theta$ ... [Adding (I) and (II)] [½]

$\therefore \quad x^2 + y^2 = r^2 (\cos^2 \theta + \sin^2 \theta)$ [½]

$\therefore \quad x^2 + y^2 = r^2 \times 1$ ... ($\because \sin^2 \theta + \cos^2 \theta = 1$) [½]

Ans. $x^2 + y^2 = r^2$ [½] [3]

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