SOLUTION

Q.1. (A) For each of the following sub-questions four alternative answers are given. Choose the correct alternative and write its alphabet: [4]

(1) If $\Delta ABC \sim \Delta DEF$ and $\angle A = 48°$, then $\angle D$ = ______________.

(a) 48° (b) 83° (c) 49° (d) 132°

(2) AP is a tangent at A drawn to the circle with centre O from an external point P. OP = 12 cm and $\angle OPA = 30°$, then the radius of the circle is ______________.

(a) 12 cm (b) $6\sqrt{3}$ cm (c) 6 cm (d) $12\sqrt{3}$ cm

(3) Seg AB is parallel to X-axis and co-ordinates of the point A are (1, 3), then the co-ordinates of the point B can be ______________.

(a) (–3, 1) (b) (5, 1) (c) (3, 0) (d) (–5, 3)

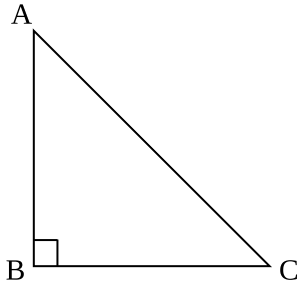
(4) The value of 2tan 45° – 2sin 30° is ______________.

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Ans. (1) - (a), (2) - (c), (3) - (d), (4) - (b)

Q.1. (B) Solve the following questions. **[4]**

(1)



In ΔABC, $\angle ABC = 90°$, $\angle BAC = \angle BCA = 45°$. If $AC = 9\sqrt{2}$, then find the value of AB.

Solution:

In ΔABC, $\angle ABC = 90°$
$\angle BAC = \angle BCA = 45°$ } .... (Given)

Let $AB = BC = x$

By Pythagoras theorem,

$AB^2 + BC^2 = AC^2$

$\therefore \quad x^2 + x^2 = (9\sqrt{2})^2$

$\therefore \quad 2x^2 = 81 \times 2$

$\therefore \quad x^2 = 81$

$\therefore \quad x = 9$

$\therefore \quad AB = 9$

(2) Chord AB and chord CD of a circle with centre O are congruent. If *m*(arc AB) = 120°, then find *m*(arc CD).

Solution:

Chord CD $\cong$ Chord AB ...(Given)

$\therefore$ *m*(arc CD) = *m*(arc AB) ...(Corresponding arcs of congruent chords)

= 120° ...[As *m*(arc AB) = 120°, given]

$\therefore$ *m*(arc CD) = 120°

(3) Find the Y co-ordinate of the centroid of a triangle whose vertices are (4, –3), (7, 5) and (–2, 1).

Solution:

Let $(4, -3) \equiv (x_1, y_1)$

$$(7, 5) \equiv (x_2, y_2)$$
$$(-2, 1) \equiv (x_3, y_3)$$

By the centroid formula,

$$y = \frac{y_1 + y_2 + y_3}{3}$$
$$= \frac{-3 + 5 + 1}{3}$$
$$= \frac{3}{3}$$
$$= 1$$

Ans. The Y co-ordinate of the centroid of the triangle is 1.

(4) If sin θ = cos θ, then what will be the measure of angle θ?

Solution:

$\sin\theta = \cos\theta$ (Given)

$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$

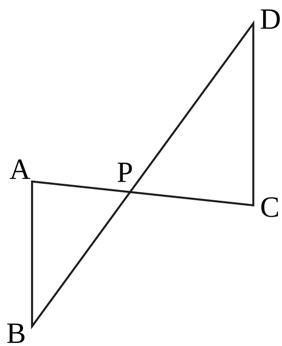
$\therefore$ $\theta = 45°$

Q.2. (A) Complete the following activities and rewrite them (any *two*): [4]

(1) In the alongside figure, seg AC and seg BD intersect each other in point P.

If $\frac{AP}{CP} = \frac{BP}{DP}$, then complete the following activity to prove

ΔABP ~ ΔCDP.



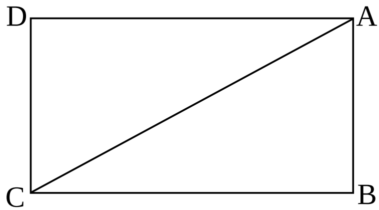
Activity: In ΔAPB and ΔCDP,

$\frac{AP}{CP} = \frac{BP}{DP}$ ...Given

$\therefore$ ∠APB ≅ ∠CPD ...Vertically opposite angles

$\therefore$ ΔABP ~ ΔCDP ...SAS test of similarity

(2) In the alongside figure, □ABCD is a rectangle. If AB = 5, AC = 13, then complete the following activity to find BC.



Activity:

ΔABC is [right-angled] triangle.

$\therefore$ By Pythagoras theorem,

$AB^2 + BC^2 = AC^2$

$\therefore$ $25 + BC^2 = \boxed{169}$

$\therefore$ $BC^2 = \boxed{144}$

$\therefore$ $BC = \boxed{12}$

(3) Complete the following activity to prove:

$\cot\theta + \tan\theta = \text{cosec}\,\theta \times \sec\theta$

Activity:

$$\text{L.H.S.} = \cot\theta + \tan\theta$$

$$= \frac{\cos\theta}{\sin\theta} + \frac{\boxed{\sin\theta}}{\cos\theta}$$

$$= \frac{\boxed{\cos^2\theta} + \sin^2\theta}{\sin\theta \times \cos\theta}$$

$$= \frac{1}{\sin\theta \times \cos\theta} \text{ .... } \boxed{\because\ \sin^2\theta + \cos^2\theta = 1}$$

$$= \frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$$

$$= \boxed{\text{cosec}\,\theta} \times \sec\theta$$

$\therefore$ L.H.S. = R.H.S.

Q.2. (B) Solve the following sub-questions (any *four*): [8]

(1) If $\Delta ABC \sim \Delta PQR$, AB:PQ = 4:5 and $A(\Delta PQR) = 125\ cm^2$, then find $A(\Delta ABC)$.

Solution:

$\Delta ABC \sim \Delta PQR$ ...(Given)

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$$ ... (Theorem of area of two similar triangles)

$$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2$$

$$\therefore \frac{A(\Delta ABC)}{125} = \left(\frac{4}{5}\right)^2$$

$$\therefore \frac{A(\Delta ABC)}{125} = \frac{16}{25}$$

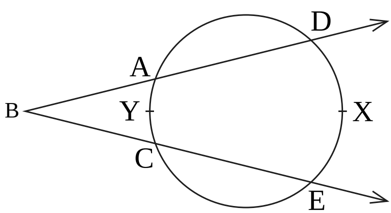
$$\therefore A(\Delta ABC) = \frac{16 \times 125}{25}$$

$$= 16 \times 5$$

$$= 80$$

$$\therefore A(\Delta ABC) = 80 \text{ sq.cm}$$

(2) In the following figure, *m*(arc DXE) = 105°, *m*(arc AYC) = 47°, then find the measure of ∠DBE.



Solution:

$$\angle DBE = \frac{1}{2}[m(\text{arc DXE}) - m(\text{arc AYC})]$$

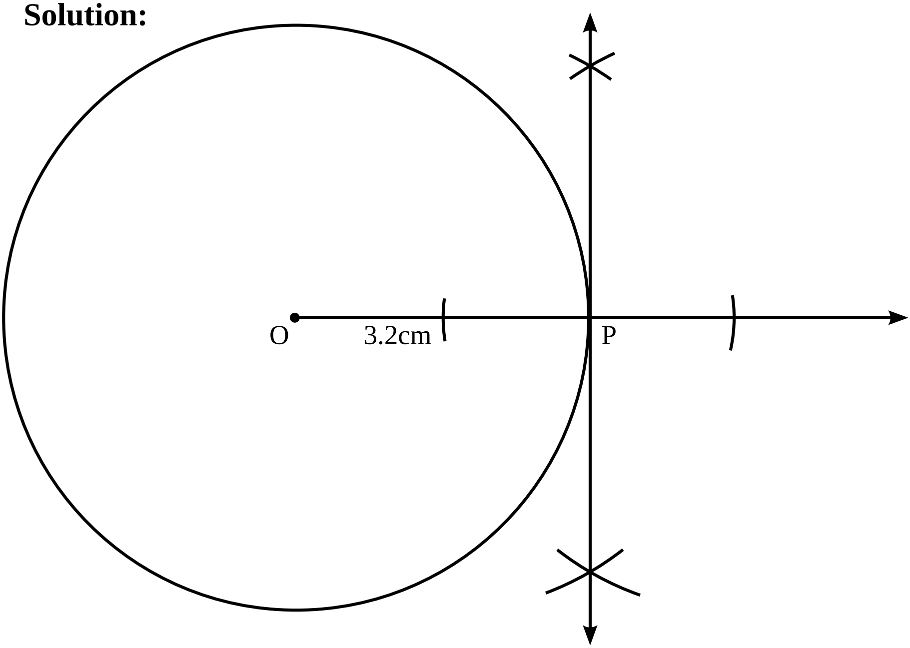
$$= \frac{1}{2}[105° - 47°]$$

$$= \frac{1}{2} \times 58°$$

$$\therefore \angle DBE = 29°$$

(3) Draw a circle of radius 3.2 cm and centre 'O'. Take any point P on it. Draw a tangent to the circle through point P using the centre of the circle.

Solution:



(4) If $\sin\theta = \frac{11}{61}$, then find the value of $\cos\theta$ using trigonometric identity.

Solution:

$$\sin^2\theta + \cos^2\theta = 1 \quad \text{...(Trigonometric identity)}$$

$$\therefore \left(\frac{11}{61}\right)^2 + \cos^2\theta = 1$$

$$\therefore \frac{121}{3721} + \cos^2\theta = 1$$

$$\therefore \cos^2\theta = 1 - \frac{121}{3721}$$

$$= \frac{3721 - 121}{3721}$$

$$= \frac{3600}{3721}$$

$$\therefore \cos\theta = \frac{60}{61} \quad \text{...(Taking square root on both sides)}$$

(5) In ΔABC, AB = 9 cm, BC = 40 cm, AC = 41 cm. State whether ΔABC is a right-angled triangle or not. Write reason.

Solution:

In ΔABC,

$AB^2 + BC^2 = 9^2 + (40)^2$

$= 81 + 1600$

$= 1681$ ...(i)

$AC^2 = (41)^2 = 1681$ ...(ii)

$\therefore$ $AB^2 + BC^2 = AC^2$ ...[From (i) and (ii)]

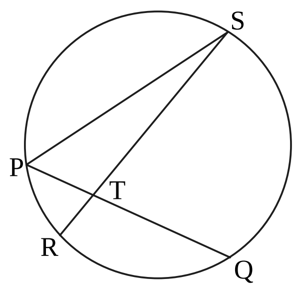
$\therefore$ ΔABC is a right-angled tringle.

(By converse of Pythagoras theorem)

Q.3. (A) Complete the following activity and rewrite it. (Any *one*): [3]

(1) In the alongside figure, chord PQ and chord RS intersect each other at point T. If $\angle STQ = 58°$ and $\angle PSR = 24°$, then complete the following activity to verify:

$\angle STQ = \frac{1}{2}[m(\text{arc PR}) + m(\text{arc SQ})]$



Activity:

In ΔPTS,

$\angle SPQ = \angle STQ -$ $\boxed{\angle PST}$ (Exterior angle theorem)

$\therefore$ $\angle SPQ = 34°$

$\therefore$ $m(\text{arc QS}) = 2 \times \boxed{34}° = 68°$ ...(Inscribed angle theorem)

Similarly, $m(\text{arc PR}) = 2\angle PSR = \boxed{48}°$

$\therefore$ $\frac{1}{2}[m(\text{arc QS}) + m(\text{arc PR})] = \frac{1}{2} \times \boxed{68 + 48}° = 58°$ ...(I)

But $\angle STQ = 58°$ ...(II), given

$\therefore$ $\frac{1}{2}[m(\text{arc PR}) + m(\text{arc QS})] = \boxed{\angle STQ}$ ...From (I) and (II)

(2) Complete the following activity to find the co-ordinates of point P which divides seg AB in the ratio 3:1 where A(4, –3) and B(8, 5).

Activity:

By section formula,

$$x = \frac{mx_2 + nx_1}{\boxed{m + n}}, \qquad y = \frac{\boxed{my_2 + ny_1}}{m + n}$$

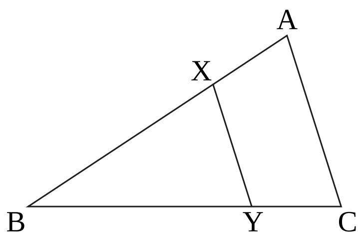
$$\therefore \quad x = \frac{3 \times 8 + 1 \times 4}{3 + 1}, \qquad y = \frac{3 \times 5 + 1 \times (-3)}{3 + 1}$$

$$= \frac{\boxed{24} + 4}{4}, \qquad = \frac{\boxed{15} - 3}{4}$$

$$\therefore \quad x = \boxed{7} \qquad \therefore \; y = \boxed{3}$$

Q.3. (B) Solve the following sub-questions (any *two*): [6]

(1) In ΔABC, seg XY || side AC. If 2AX = 3BX and XY = 9, then find the value of AC.



Solution:

In ΔABC,

$$\left.\begin{array}{r} \text{seg XY} \parallel \text{side AC} \\ 2AX = 3BX \end{array}\right\} \quad \text{...(Given)}$$

$$\therefore \quad \frac{AX}{BX} = \frac{3}{2}$$

$$\therefore \quad \frac{AX + BX}{BX} = \frac{3 + 2}{2} \quad \text{...(By Componendo)}$$

$$\therefore \quad \frac{AB}{BX} = \frac{5}{2} \quad \text{...(i) } (\because A\text{–}X\text{–}B)$$

In ΔBCA and ΔBYX,

$$\angle BCA \cong \angle BYX \quad \text{...(Corresponding angles)}$$

$$\angle CBA \cong \angle YBX \quad \text{...(Common angle)}$$

$$\therefore \quad \Delta BCA \cong \Delta BYX \quad \text{...(AA test of similarity)}$$

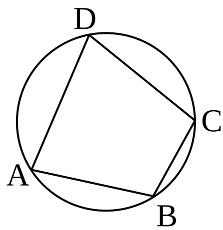
$$\therefore \quad \frac{AB}{BX} = \frac{AC}{XY} \quad \text{...(C.S.S.T)}$$

$$\therefore \quad \frac{5}{2} = \frac{AC}{9}$$

$$\therefore \quad AC = \frac{5 \times 9}{2}$$

$$\therefore \quad AC = 22.5$$

(2) Prove that "Opposite angles of cyclic quadrilateral are supplementary."



Solution:

Given: □ABCD is cyclic.

To prove:: $\angle B + \angle D = 180°$

$\angle A + \angle C = 180°$

Proof:

Arc ABC is intercepted by the inscribed angle ADC.

$$\therefore \quad \angle ADC = \frac{1}{2} m(\text{arc ABC}) \quad \text{...(i)}$$

Similarly, arc ADC is intercepted by the insctribed angle ABC.

$$\therefore \quad \angle ABC = \frac{1}{2} m(\text{arc ADC}) \quad \text{...(ii)}$$

$$\therefore \quad m\angle ADC + m\angle ABC = \frac{1}{2} m(\text{arc ABC}) + \frac{1}{2} m(\text{arc ADC})$$

[From (i) and (ii)]

$$= \frac{1}{2} [m(\text{arc ABC}) + m(\text{arc ADC})]$$

$$= \frac{1}{2} \times 360° \quad \text{...(Arcs ABC and ADC constitute a complete circle)}$$

$$\therefore \quad \angle ADC + \angle ABC = 180°$$

Similarly, we can prove that

$$\angle A + \angle C = 180°.$$

(3) ΔABC ~ ΔPQR. In ΔABC, AB = 5.4 cm, BC = 4.2 cm, AC = 6.0 cm, AB:PQ = 3:2, then construct ΔABC and ΔPQR.

Solution:

$\Delta ABC \sim \Delta PQR$

$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ...(C.S.S.T)

$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{3}{2}$ ...(AB:PQ = 3:2, given)

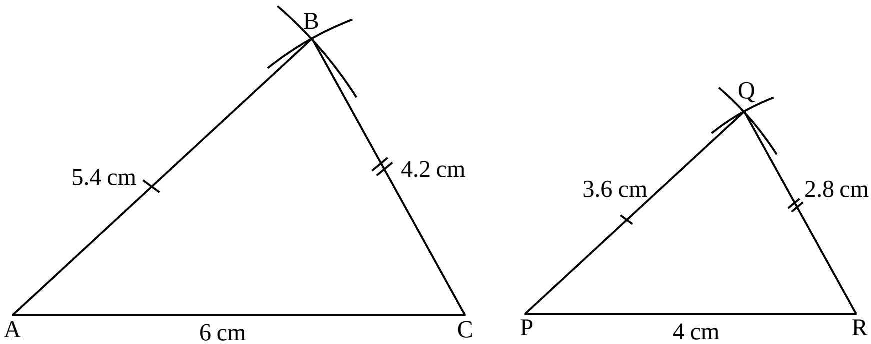
$\therefore \quad \frac{5.4}{PQ} = \frac{4.2}{QR} = \frac{6}{PR} = \frac{3}{2}$

$\therefore \quad \frac{5.4}{PQ} = \frac{3}{2}$

$\therefore \quad PQ = \frac{5.4 \times 2}{3} = 3.6 \text{ cm}$

$\frac{4.2}{QR} = \frac{3}{2}$

$\therefore \quad QR = \frac{4.2 \times 2}{3} = 2.8 \text{ cm}$



(4) Show that:

$$\frac{\tan A}{(1+\tan^2 A)^2} + \frac{\cot A}{(1+\cot^2 A)^2} = \sin A \times \cos A$$

Solution:

$$\text{LHS} = \frac{\tan A}{(1+\tan^2 A)^2} + \frac{\cot A}{(1+\cot^2 A)^2}$$

$$= \frac{\tan A}{(\sec^2 A)^2} + \frac{\cot A}{(\text{cosec}^2 A)^2} \quad ...(\because 1+\tan^2\theta = \sec^2\theta \text{ and } 1+\cot^2\theta = \text{cosec}^2\theta)$$

$$= \frac{\sin A}{\cos A} \times (\cos^2 A)^2 + \frac{\cos A}{\sin A} \times (\sin^2 A)^2$$

$$= \sin A \times \cos^3 A + \cos A \times \sin^3 A$$

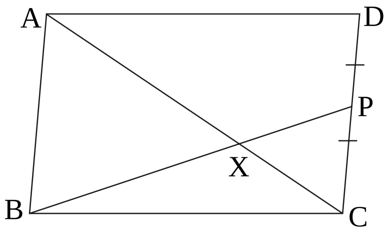
$$= \sin A.\cos A \ (\cos^2 A + \sin^2 A)$$

$$= \sin A.\cos A \qquad ...(\because \ \sin^2 \theta. \cos^2 \theta = 1)$$

$\therefore$ LHS = RHS

Q.4. Solve the following sub-questions (any *two*) **[8]**

(1)



□ABCD is a parallelogram. Point P is the midpoint of side CD. Seg BP intersects diagonal AC at point X, then prove that: 3AX = 2AC

Solution:

Proof: □ABCD is parallelogram, and point P is the midpoint of side DC. ...(Given)

$\therefore$ AB = CD = 2CP ...(i)

Now, in ΔAXB and ΔCXP,

$\angle AXB \cong \angle CXP$ ...(Vertically opposite angles)

$\angle BAX \cong \angle PCX$ ...(Alternate angles)

$\therefore$ $\Delta AXB \sim \Delta CXP$ ...(By AA test)

$\therefore$ $\frac{AX}{CX} = \frac{AB}{CP}$ ...(C.S.S.T)

$\therefore$ $\frac{AX}{CX} = \frac{2CP}{CP}$ ...[From (i)]

$\therefore$ $\frac{AX}{CX} = \frac{2}{1}$

$\therefore$ $CX = \frac{AX}{2}$

Now, AC = AX + CX ...(A–X–C)

$\therefore$ $AC = AX + \frac{AX}{2}$

$\therefore \quad AC = \frac{2AX + AX}{2}$

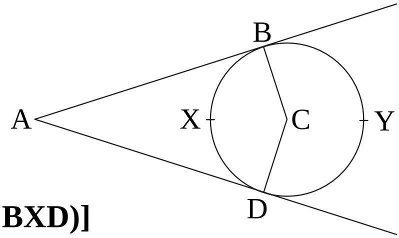
$\therefore \quad 2AC = 3AX$

$\therefore \quad 3AX = 2AC$

Hence proved.

(2) In the alongside figure, seg AB and seg AD are tangent segments drawn to a circle with centre C from exterior point A, then prove that:

$\angle A = \frac{1}{2}$ [*m*(arc BYD) – *m*(arc BXD)]



Solution:

seg CB ⊥ seg AB
seg CD ⊥ seg AD } (Tangent perpendicular to radius)

$\therefore \quad \angle ABC = \angle ADC = 90°$ ...(i)

Now, in □ABCD,

$\angle A + \angle C + \angle B + \angle D = 360°$ ...(Sum of angles of quadrilateral)

$\therefore \quad \angle A + \angle C + 90° + 90 = 360°$ ...[From (i)]

$\therefore \quad \angle A + \angle C = 360° - 180°$

$\therefore \quad \angle A + \angle C = 180°$

$\therefore \quad \angle A = 180° - \angle C$

But $\angle C = m(\text{arc BXD})$ ....(Definition of measure of arc)

$\therefore \quad \angle A = 180° - m(\text{arc BXD})$ ...(ii)

Now,

$m(\text{arc BXD}) + m(\text{arc BYD}) = 360°$

...(Measure of complete circle)

$\therefore \quad \frac{1}{2} m(\text{arc BXD}) + \frac{1}{2} m(\text{arc BYD}) = 180°$ ...(iii)

...[Multiplying (ii) by $\frac{1}{2}$]

$\therefore \quad \angle A = \frac{1}{2} m(\text{arc BXD}) + \frac{1}{2} m(\text{arc BYD}) - m(\text{arc BXD})$

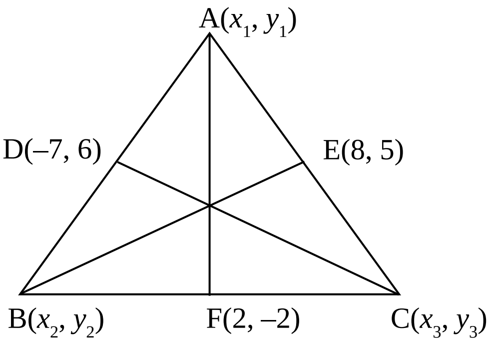
...[From (ii) and (iii)]

$= \frac{1}{2} m(\text{arc BYD}) - \frac{1}{2} m(\text{arc BXD})$

$\therefore \quad \angle A = \frac{1}{2} [m(\text{arc BYD}) - m(\text{arc BXD})]$

(3) **Find the co-ordinates of centroid of a triangle if points D(–7, 6), E(8, 5) and F(2, –2) are the midpoints of the sides of that triangle.**

Solution:



Let $A \equiv (x_1, y_1)$, $B \equiv (x_2, y_2)$, $C \equiv (x_3, y_3)$

By the midpoint formula,

$$\frac{x_1 + x_2}{2} = -7$$

$\therefore$ $x_1 + x_2 = -14$ ...(i)

$$\frac{y_1 + y_2}{2} = 6$$

$\therefore$ $y_1 + y_2 = 12$ ...(ii)

$$\frac{x_2 + x_3}{2} = 2$$

$\therefore$ $x_2 + x_3 = 4$ ...(iii)

$$\frac{y_2 + y_3}{2} = -2$$

$\therefore$ $y_2 + y_3 = -4$ ...(iv)

$$\frac{x_1 + x_3}{2} = 8$$

$\therefore$ $x_1 + x_3 = 16$ ...(v)

$$\frac{y_1 + y_3}{2} = 5$$

$\therefore$ $y_1 + y_3 = 10$ ...(vi)

Adding equations (i), (iii) and (v),

$$2x_1 + 2x_2 + 2x_3 = 6$$

$\therefore \quad x_1 + x_2 + x_3 = 3$

$\therefore \quad \dfrac{x_1 + x_2 + x_3}{3} = \dfrac{3}{3}$

$\therefore \quad \dfrac{x_1 + x_2 + x_3}{3} = 1$

Adding equations (ii), (iv) and (vi),

$y_1 + y_2 + y_3 = 9$

$\therefore \quad \dfrac{y_1 + y_2 + y_3}{3} = 3$

But $G \equiv \left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3} \right)$ ...(Centroid formula)

$\therefore \quad G \equiv (1, 3)$

Q.5. Solve the following sub-question (any *one*): [3]

(1) *a* and *b* are natural numbers and *a* > *b*. If ($a^2 + b^2$), ($a^2 - b^2$) and 2*ab* are the sides of a triangle, then prove that the triangle is right angled.

Find out two Pythagorean triplets by taking suitable values of *a* and *b*.

Solution:

a and *b* are natural numbers and $a > b$. ...(Given)

Longest side = $(a^2 + b^2)$

$(a^2 + b^2)^2 = a^4 + 2a^2b^2 + b^4$ ...(i)

and

$(a^2 - b^2)^2 + (2ab)^2 = a^4 - 2a^2b^2 + b^4 + 4a^2b^2$

$\therefore \quad (a^2 - b^2)^2 + (2ab)^2 = a^4 + 2a^2b^2 + b^4$ ...(ii)

$\therefore \quad (a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$ ...[From (i) and (ii)]

$\therefore$ By converse of Pythagoras theorem,

$(a^2 + b^2)$, $(a^2 - b^2)$ and $2ab$ are the sides of a right-angled triangle.

Now, if $a = 2$ and $b = 1$ then

$a^2 + b^2 = 2^2 + 1^2 = 4 + 1 = 5$

$(a^2 - b^2) = 2^2 - 1^2 = 4 - 1 = 3$

$2ab = 2 \times 2 = 4$

$\therefore$ (3, 4, 5) is a Pythagorean triplet.

Similarly, if $a = 3$ and $b = 2$, then

$a^2 + b^2 = 3^2 + 2^2 = 9 + 4 = 13$

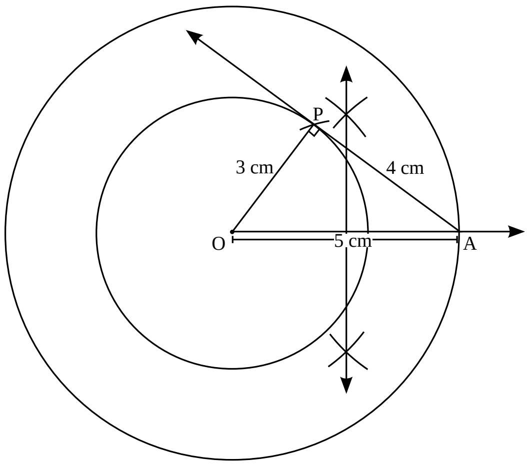
$a^2 - b^2 = 3^2 - 2^2 = 9 - 4 = 5$

$2ab = 2 \times 3 \times 2 = 12$

$\therefore$ (5, 12, 13) is a Pythagorean triplet.

(2) Construct two concentric circles with centre O and radii 3 cm and 5 cm. Construct a tangent to the smaller circle from any point A on the larger circle. Measure and wirte the length of the tangent segment. Calculate the length of the tangent segment using Pythagoras theorem.

Solution:



ΔOAP is a right angled triangle.

OA = 5 cm, OP = 3 cm, AP = ?

By Pythagoras theorem,

$$AP^2 = OA^2 - OP^2$$
$$= 5^2 - 3^2$$
$$= 25 - 9$$
$$= 16$$

$\therefore$ AP = 4

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