SOLUTION

Q.1. (A) Four alternative answers are given for every sub-question. Select the correct alternative and write the alphabet of that answer. [4]

(1) Out of the following which is the Pythagorean triplet?

(a) (1, 5, 10) (b) (3, 4, 5) (c) (2, 2, 2) (d) (5, 5, 2)

(2) Two circles of radii 5.5 cm and 3.3 cm respectively touch each other externally. What is the distance between their centres?

(a) 4.4 cm (b) 2.2 cm (c) 8.8 cm (d) 8.9 cm

(3) Distance of point (–3, 4) from the origin is ....

(a) 7 (b) 1 (c) –5 (d) 5

(4) Find the volume of a cube of side 3 cm:

(a) 27 cm^3 (b) 9 cm^3 (c) 81 cm^3 (d) 3 cm^3

Ans. (1) - (b), (2) - (c), (3) - (d), (4) - (a).

Q.1. (B) Solve the following questions. [4]

(1) The ratio of corresponding sides of similar triangle is 3 : 5, then find the ratio of their areas.

Solution:

Let the Δ_1 and Δ_2 have sides S_1 and S_2 respectively.

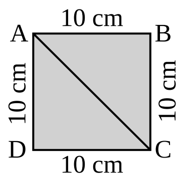
$$\frac{A(\Delta_1)}{A(\Delta_2)} = \frac{(S_1)^2}{(S_2)^2} \quad \text{... (Ratio of areas of two similar triangles)}$$

$$\therefore \quad \frac{A(\Delta_1)}{A(\Delta_2)} = \frac{3^2}{5^2} = \frac{9}{25}$$

Ans. The ratio of the areas of given triangles is 9:25.

(2) Find the diagonal of a square whose side is 10 cm.

Solution:



□ABCD is a square with side 10 cm.

In ΔABC, ∠B = 90° ...(Property of a square)

$\therefore$ $AC^2 = AB^2 + BC^2$ ...(Pythagoras theorem)

$\therefore$ $AC^2 = 10^2 + 10^2$

$= 100 + 100$

$= 200$

$\therefore$ $AC = 10\sqrt{2}$

Ans. The diagonal of a square is $10\sqrt{2}$ cm.

(3) □ABCD is cyclic. If ∠B = 110°, then find measure of ∠D.

Solution:

∠B = 110° ...(Given)

$\therefore$ ∠B + ∠D = 180° ...(Opposite angles of a cyclic quadrilateral)

$\therefore$ 110° + ∠D = 180°

$\therefore$ ∠D = 180° – 110°

$\therefore$ ∠D = 70°

Ans. The measure of ∠D = 70°.

(4) Find the slope of the line passing through the points A(2, 3) and B(4, 7).

Solution:

$A(2, 3) = (x_1, y_1)$

$B(4, 7) = (x_2, y_2)$

$$\text{Slope of the line AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{7-3}{4-2}$$

$$= \frac{4}{2}$$

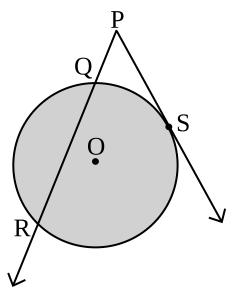
$$= 2$$

Ans. The slope of the line AB is 2.

Q.2. (A) Complete and write the following activities. (Any two) [4]

(1) In the adjoining figure, 'O' is the centre of the circle, seg PS is a tangent segment and S is the point of contact. Line PR is a secant.

If PQ = 3.6, QR = 6.4, find PS.



Solution:

PS^2 = PQ x ☐ ...(Tangent secant segments theorem)

= PQ x (PQ + ☐)

= 3.6 x (3.6 + 6.4)

= 3.6 x ☐

= 36

∴ PS = ☐ ... (By taking square roots)

Solution:

PS^2 = PQ x PR ...(Tangent secant segments theorem)

= PQ x (PQ + QR)

$= 3.6 \times (3.6 + 6.4)$

$= 3.6 \times \boxed{10}$

$= 36$

$\therefore \quad PS = \boxed{6}$ ... (By taking square roots)

Ans. PS = 6.

(2) If $\sec\theta = \frac{25}{7}$, find the value of $\tan\theta$.

Solution: $1 + \tan^2\theta = \sec^2\theta$

$\therefore \quad 1 + \tan^2\theta = \left(\frac{25}{7}\right)^{\square}$

$\therefore \quad \tan^2\theta = \frac{625}{49} - \square$

$= \frac{625 - 49}{49}$

$= \frac{\square}{49}$

$\therefore \quad \tan\theta = \frac{\square}{7}$ ...(By taking square roots)

Solution: $1 + \tan^2\theta = \sec^2\theta$

$\therefore \quad 1 + \tan^2\theta = \left(\frac{25}{7}\right)^{\boxed{2}}$

$\therefore \quad \tan^2\theta = \frac{625}{49} - \boxed{1}$

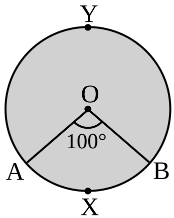
$= \frac{625 - 49}{49}$

$= \frac{\boxed{576}}{49}$

$\therefore \quad \tan\theta = \frac{\boxed{24}}{7}$ ...(By taking square roots)

Ans. $\tan\theta = \frac{24}{7}$

(3) In the given figure, O is the centre of the circle. Using given information complete the following table.



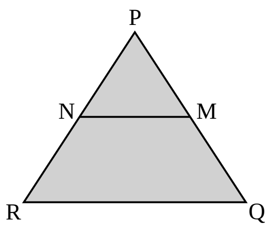
Type of arc	**Name of the arc**	**Measure of the arc**
Minor arc		
Major arc		

Solution:

Type of arc	**Name of the arc**	**Measure of the arc**
Minor arc	AXB	100°
Major arc	AYB	260°

Q.2. (B) Solve the following sub-questions. (Any four) [8]

(1) **In ΔPQR, NM ∥ RQ. If PM = 15, MQ = 10, NR = 8, then find PN.**



Solution:

In ΔPQR, NM || QR.

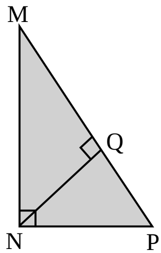
$\therefore \quad \frac{PN}{NR} = \frac{PM}{MQ}$ ...(BPT)

$\therefore \quad \frac{PN}{8} = \frac{15}{10}$

$\therefore \quad PN = \frac{15 \times 8}{10}$

Ans.PN = 12 unit

(2)



In ΔMNP. ∠MNP = 90°, seg NQ ⊥ seg MP. If MQ = 9, QP = 4, then find NQ.

Solution:

In right angled triangle MNP, NQ ⊥ MP.

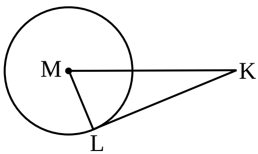
∴ $NQ^2 = MQ \times QP$ ...(Property of geometric mean)

∴ $NQ^2 = 9 \times 4$

∴ $NQ = 3 \times 2$ ...(Taking square roots)

Ans. NQ = 6

(3)



In the given figure, M is the centre of the circle and seg KL is a tangent segment. L is a point of contact. If MK = 12, KL = $6\sqrt{3}$, then find the radius of the circle.

Solution:

ML is a radius and KL is a tangent.

∴ ∠MLK = 90° ...(Radius is perpendicular to the tangent at the point of contact)

In ΔMLK, ∠MLK = 90°.

∴ $MK^2 = ML^2 + KL^2$ ...(Pythagoras theorem)

∴ $12^2 = ML^2 + (6\sqrt{3})^2$

∴ $144 = ML^2 + 108$

∴ $ML^2 = 144 - 108$

∴ $ML^2 = 36$

∴ ML = 6 unit ...(Taking square root)

Ans. The radius of the circle is 6 unit.

(4) Find the co-ordinates of midpoint of the segment joining the points (22, 20) and (0, 16).

Solution:

Let (22, 20) = (x_1, y_1)

(0, 16) = (x_2, y_2)

By midpoint formula,

$x = \dfrac{x_2 + x_1}{2}$ and $y = \dfrac{y_2 + y_1}{2}$

$\therefore$ $x = \dfrac{0 + 22}{2}$ and $y = \dfrac{16 + 20}{2}$

$\therefore$ $x = \dfrac{22}{2}$ and $y = \dfrac{36}{2}$

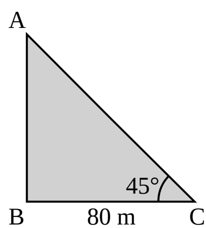
$\therefore$ $x = 11$ and $y = 18$

Ans. (11, 18) is the midpoint of the segment joining the points (22, 20) and (0, 16).

(5) A person is standing at a distance of 80 metres from a Church and looking at its top. The angle of elevation is of 45°. Find the height of the Church.

Solution:

Let AB be the Church and the person is standing at C.



$\tan\theta = \dfrac{AB}{BC}$

$\therefore$ $\tan 45° = \dfrac{AB}{80}$

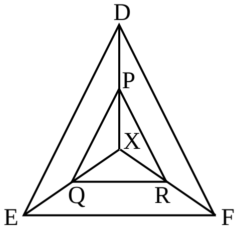
$\therefore$ $1 = \dfrac{AB}{80}$

$\therefore$ AB = 80 m

Ans. The height of the Church is 80 m.

Q.3. (A) Complete and write the following activities. (Any one) **[3]**

(1)



In the given figure, X is any point in the interior of the triangle. Point X is joined to the vertices of triangle.
Seg PQ ∥ seg DE, seg QR ∥ seg EF. Complete the activity and prove that seg PR ∥ seg DF.

Proof:

In ΔXDE,

PQ ∥ DE ...(Given)

$\therefore \frac{XP}{PD} = \frac{\square}{QE}$ ...(Basic proportionality theorem) ...(i)

In ΔXEF,

QR ∥ EF ...(Given)

$\therefore \frac{XQ}{\square} = \frac{XR}{\square}$ ...($\square$) ...(ii)

$\therefore \frac{XP}{PD} = \frac{\square}{\square}$ ...[From (i) and (ii)]

∴ seg PR ∥ seg DF ...(By converse of basic proportionality theorem)

Solution:

Proof:

In ΔXDE,

PQ ∥ DE ...(Given)

$\therefore \frac{XP}{PD} = \frac{\boxed{XQ}}{QE}$ ...(Basic proportionality theorem) ...(i)

In ΔXEF,

QR ∥ EF ...(Given)

$\therefore \frac{XQ}{\boxed{QE}} = \frac{XR}{\boxed{RF}}$ ...($\boxed{\text{Basic proportionality theorem}}$) ...(ii)

$\therefore \quad \frac{XP}{PD} = \frac{\boxed{XR}}{\boxed{RF}}$ ...[From (i) and (ii)]

$\therefore$ seg PR ∥ seg DF ...(By converse of basic proportionality theorem)

(2) If A(6, 1), B(8, 2), C(9, 4) and D(7, 3) are the vertices of □ABCD, show that □ABCD is a parallelogram.

Solution:

Slope of line $= \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ Slope of line AB $= \frac{2-1}{8-6} = \square$ ...(i)

$\therefore$ Slope of line BC $= \frac{4-2}{9-8} = \square$ ...(ii)

$\therefore$ Slope of line CD $= \frac{3-4}{7-9} = \square$ ...(iii)

$\therefore$ Slope of line DA $= \frac{3-1}{7-6} = \square$ ...(iv)

$\therefore$ Slope of line AB $= \square$ ...[From (i) and (iii)]

$\therefore$ Line AB ∥ line CD

$\therefore$ Slope of line BC $= \square$ ...[From (ii) and (iv)]

$\therefore$ Line BC ∥ line DA

Both the pairs of opposite sides of the quadrilateral are parallel.

$\therefore$ □ABCD is a parallelogram.

Solution:

Slope of line $= \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ Slope of line AB $= \frac{2-1}{8-6} = \boxed{\frac{1}{2}}$ ...(i)

$\therefore$ Slope of line BC $= \frac{4-2}{9-8} = \boxed{2}$ ...(ii)

$\therefore$ Slope of line CD $= \frac{3-4}{7-9} = \boxed{\frac{1}{2}}$ ...(iii)

$\therefore$ Slope of line DA = $\frac{3-1}{7-6}$ = $\boxed{2}$ ...(iv)

$\therefore$ Slope of line AB = Slope of line CD ...[From (i) and (iii)]

$\therefore$ Line AB ∥ line CD

$\therefore$ Slope of line BC = Slope of line DA ...[From (ii) and (iv)]

$\therefore$ Line BC ∥ line DA

Both the pairs of opposite sides of the quadrilateral are parallel.

$\therefore$ □ABCD is a parallelogram.

Q.3. (B) Solve the following sub-questions. (Any two) [6]

(1) In ΔPQR, point S is the mid-point of side QR. If PQ = 11, PR = 17, PS = 13, find QR.

Solution:

PS is a median in ΔPQR.

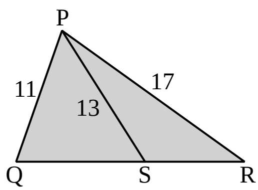
$\therefore$ QS = SR ...(i) and

$\therefore$ $PQ^2 + PR^2 = 2PS^2 + 2QS^2$ ...(Apolloneous theorem)

$\therefore$ $11^2 + 17^2 = 2[(13)^2 + QS^2]$

$\therefore$ $121 + 289 = 2(169 + QS^2)$

$\therefore$ $169 + QS^2 = \frac{410}{2}$



$\therefore$ $QS^2 = 205 - 169$

$\therefore$ $QS^2 = 36$

$\therefore$ QS = 6 ...(Taking square root)

$\therefore$ SR = 6 ...[From (i)]

QR = QS + SR ...(Q–S–R)

$\therefore$ QR = 6 + 6

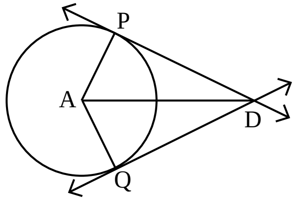
$\therefore$ QR = 12 unit

Ans. QR = 12 unit.

(2) Prove that, tangent segments drawn from an external point to the circle are congruent.

Solution:

Given: A is the centre of the circle and D is a point in the exterior of the circle. P and Q are the points of contact of the two tangents from D to the circle.



To prove: seg DP ≅ seg DQ

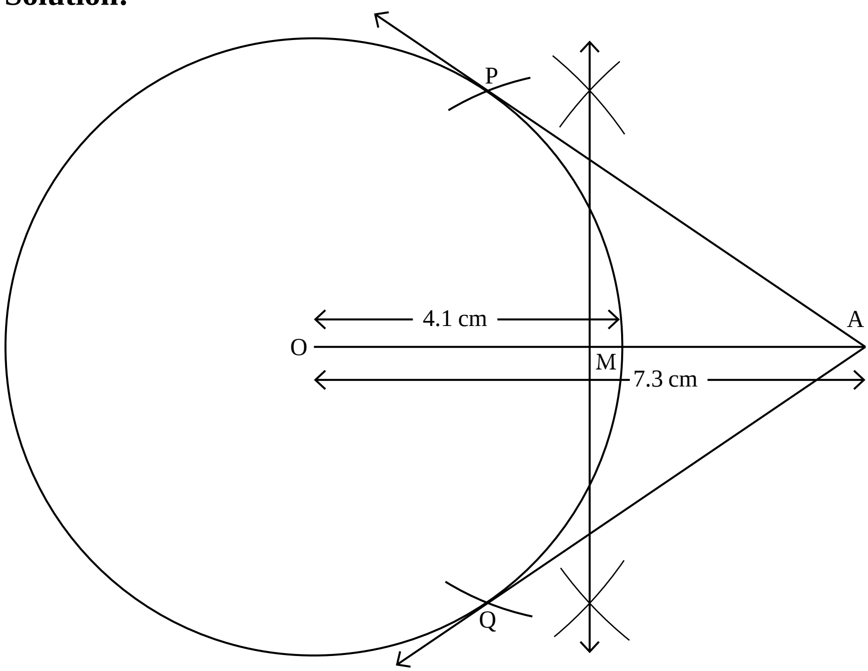
Construction: Join A and D.

Proof: In ΔPAD and ΔQAD,

	seg PA ≅ seg QA	...(Radii of the same circle)
	seg AD ≅ seg AD	...(Common side)
	∠APD ≅ ∠AQD = 90°	...(Tangent theorem)
∴	ΔPAD ≅ ΔQAD	...(Hypotenuse-side test)
∴	seg DP ≅ seg DQ	...(c.s.c.t)
	Hence, proved.	

(3) Draw a circle with radius 4.1 cm. Construct tangents to the circle from a point at a distance 7.3 cm from the centre.

Solution:



AP and AQ are the required tangents on the circle with centre O from point A at a distance of 7.3 cm from the centre.

(4) A metal cuboid of measures 16 cm x 11 cm x 10 cm was melted to make coins. How many coins were made, if the thickness and diameter of each coin was 2 mm and 2 cm respectively?

Solution:

Let the measures of the metal cuboid be l = 16 cm = 160 mm, b = 11 cm = 110 mm and h = 10 cm = 100 mm.

Volume of the metal cuboid = $l \times b \times h$ ...(Formula)

$= 160 \times 110 \times 100$ ...(i)

Thickness of the coin (h) = 2 mm and

Diameter = 2 cm, so radius = 1 cm = 10 mm

Volume of the cylindrical coin = $\pi r^2 \cdot h$

$= \frac{22}{7} \times 100 \times 2$ ...(ii)

$$\text{Number of coins} = \frac{\text{Volume of the metalic cuboid}}{\text{Volume of the cylindrical coin}}$$

$$= \frac{160 \times 110 \times 100}{\frac{22}{7} \times 100 \times 2} \quad \text{...[From (i) and (ii)]}$$

$$= \frac{160 \times 110 \times 5 \times 7}{220}$$

$$= \frac{616000}{220}$$

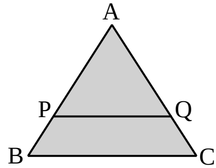
$$= 2800$$

Ans. 2800 coins were made.

Q.4. Solve the following sub-questions. (Any two) [8]

(1) In ΔABC, PQ is a line segment intersecting AB at P and AC at Q such that seg PQ $\parallel$ seg BC. If PQ divides ΔABC into two equal parts having equal areas, find $\frac{BP}{AB}$.

Solution:



$A(\Delta APQ) = A(\square PBCQ)$ ... (Given)

$\therefore \dfrac{A(\Delta APQ)}{A(\square PBCQ)} = \dfrac{1}{1}$

$\therefore \dfrac{A(\square PBCQ)}{A(\Delta APQ)} = \dfrac{1}{1}$ ...(By invertendo)

$\therefore \dfrac{A(\square PBCQ) + A(\Delta APQ)}{A(\Delta APQ)} = \dfrac{1+1}{1}$ ...(By componendo)

$\therefore \dfrac{A(\Delta ABC)}{A(\Delta APQ)} = \dfrac{2}{1}$

$\therefore \dfrac{A(\Delta APQ)}{A(\Delta ABC)} = \dfrac{1}{2}$ ...(i) ...(By invertendo)

In ΔAPQ and ΔABC,

$PQ \parallel BC$ and AB is a transversal.

$\therefore \angle APQ \cong \angle ABC$ ...(Corresponding angles)

Similarly, $\angle AQP \cong \angle ACB$

$\therefore \Delta APQ \sim \Delta ABC$ ...(By AA test)

$\therefore \dfrac{A(\Delta APQ)}{A(\Delta ABC)} = \dfrac{AP^2}{AB^2}$ ...(Ratio of areas of two similar triangles)

$\therefore \dfrac{AP^2}{AB^2} = \dfrac{1}{2}$ ...[From (i)]

$\therefore \dfrac{AP}{AB} = \dfrac{1}{\sqrt{2}}$ ...(ii) ...(Taking square roots)

$\therefore AP = \dfrac{AB}{\sqrt{2}}$ ...(iii)

From (ii),

$\dfrac{AP}{AB} = \dfrac{1}{\sqrt{2}}$

$\therefore \dfrac{AB}{AP} = \dfrac{\sqrt{2}}{1}$ ...(By invertendo)

$\therefore \dfrac{AB - AP}{AP} = \dfrac{\sqrt{2} - 1}{1}$ ...(By dividendo)

$\therefore \quad \frac{BP}{AP} = \frac{\sqrt{2} - 1}{1}$

$\therefore \quad AP = \frac{BP}{\sqrt{2} - 1}$ ...(iv)

$\therefore \quad \frac{BP}{\sqrt{2} - 1} = \frac{AB}{\sqrt{2}}$ ...[From (iii) and (iv)]

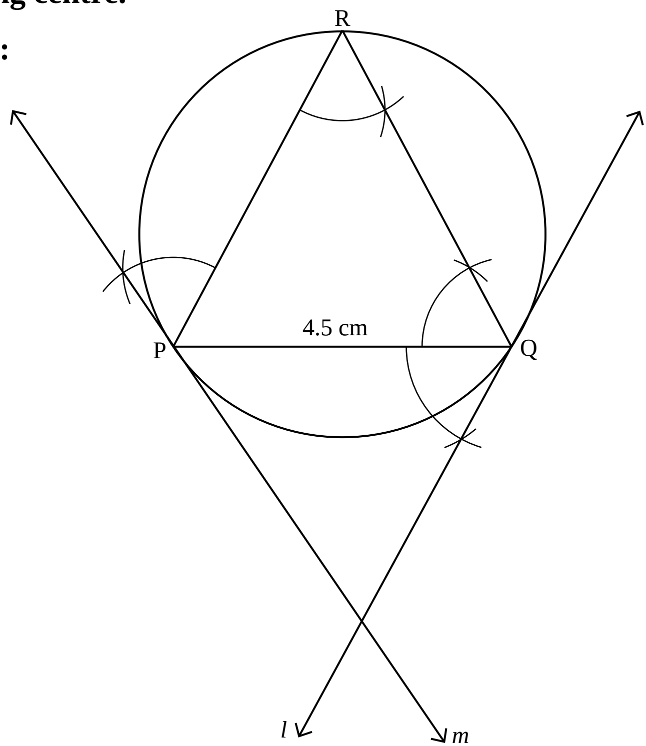
$\therefore \quad \frac{BP}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$ ...(By alternendo)

$\therefore \quad \frac{BP}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$

Ans. $\mathbf{\frac{BP}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}}$

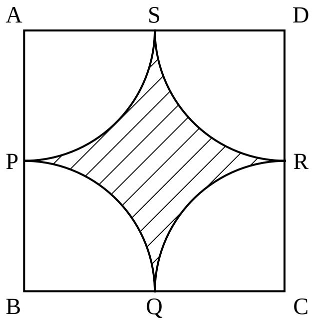
(2) Draw a circle of radius 2.7 cm and draw a chord PQ of length 4.5 cm. Draw tangents at points P and Q without using centre.

Solution:



Line *l* and line *m* are the required tangents.

(3)



In the given figure, □ABCD is a square of side 50 m. Points P, Q, R, S are midpoints of side AB, side BC, side CD, side AD respectively. Find area of shaded region.

Solution:

A(□ABCD) = $(\text{side})^2$

= $(50)^2$

= 2500 m^2

P is midpoint of AB. ...(Given)

∴ AP = BP = 25 m.

∠BAD = 90°

A(sector A-PS) $= \frac{\theta}{360} \times \pi r^2$

$= \frac{90}{360} \times \frac{22}{7} \times 25 \times 25$

$= \frac{1}{4} \times \frac{22}{7} \times 625$

= 491.07 m^2

Similarly, we can find

A(sector B-PQ) = A(sector C-QR) = A(sector D-RS)

= 491.07 m^2

A(shaded region) = A(□ABCD) – 4[A(four sectors)]

= 2500 – 4 x 491.07

= 2500 – 1964.28

= 535.72 m^2

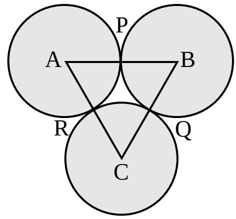
Ans. The area of the shaded region is 535.72 m^2.

[**Note:** If you take π = 3.14, the area of the shaded region will be 537.5 m^2]

Q.5. Solve the following sub-question. (Any one) **[3]**

(1) Circle with centres A, B and C touch each other externally. If AB = 3 cm, BC = 3 cm, CA = 4 cm, then find the radii of each circle.

Solution:



Let P, Q, R be the point of contacts of three externally touching circles.

Let AP = AR = x

∴ BP = BQ = 3 – x and

CR = CQ = 4 – x

Perimeter of ΔABC = 3 + 3 + 4

∴ $x + x + 3 - x + 3 - x + 4 - x + 4 - x = 10$

∴ $14 - 2x = 10$

∴ $-2x = 10 - 14$

∴ $x = \frac{-4}{-2}$

∴ $x = 2$ cm

Ans. Radius of the circle with centre A = x = 2 cm.

Radius of the circle with centre B = 3 – x = 3 – 2 = 1 cm.

Radius of the circle with centre C = 4 – x = 4 – 2 = 2 cm.

(2) If $\sin\theta + \sin^2\theta = 1$,
show that: $\cos^2\theta + \cos^4\theta = 1$.

Solution:

$\sin\theta + \sin^2\theta = 1$ ...(Given)

∴ $\sin\theta = 1 - \sin^2\theta$

∴ $\sin\theta = \cos^2\theta$ ...(∵ $1 - \sin^2\theta = \cos^2\theta$)

∴ $\sin^2\theta = \cos^4\theta$ ...(Squaring both the sides)

∴ $1 - \cos^2\theta = \cos^4\theta$

∴ **$\cos^2\theta + \cos^4\theta = 1$**

Hence proved

★★★