

SOLUTION

Q.1. (A) For each of the following sub-questions four alternative answers are given. Choose the correct alternative and write its alphabet. [4]

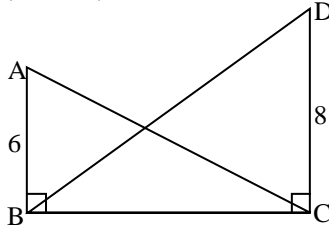
- (1) The volume of a cube of side 10 cm is [1]
(a) 1 cm^3 (b) 10 cm^3 (c) 100 cm^3 (d) 1000 cm^3
- (2) A line makes an angle of 30° with positive direction of X-axis, then the slope of the line is [1]
(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$
- (3) $\angle ACB$ is inscribed in arc ACB of a circle with centre O. If $\angle ACB = 65^\circ$, find $m(\text{arc ACB})$: [1]
(a) 65° (b) 130° (c) 295° (d) 230°
- (4) Find the perimeter of a square if its diagonal is $10\sqrt{2}$ cm: [1]
(a) 10 cm (b) $40\sqrt{2}$ cm (c) 20 cm (d) 40 cm

Ans. (1) – (d), (2) – (c), (3) – (d), (4) – (d) [4]

(B) Solve the following sub-questions.

[4]

- (1) In the following figure, $\angle ABC = \angle DCB = 90^\circ$, $AB = 6$,
 $DC = 8$, then $\frac{A(\triangle ABC)}{A(\triangle DCB)} = ?$



Solution:

$$\frac{A(\triangle ABC)}{A(\triangle DCB)} = \frac{AB}{DC}$$
$$= \frac{6}{8}$$

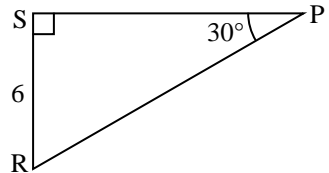
(Ratio of the areas of two triangles with a common base) [1/2]

Ans.

$$= \frac{3}{4}$$

[1/2] [1]

- (2) In the figure alongside, find the length of RP using the information given in $\triangle PSR$.



Solution:

In $\triangle PSR$, $\angle P = 30^\circ$ and $\angle S = 90^\circ$

$\therefore \angle R = 60^\circ$ (Remaining angle)

$\therefore \triangle PSR$ is $30^\circ-60^\circ-90^\circ$ triangle.

By $30^\circ-60^\circ-90^\circ$ theorem,

side opposite the 30° angle $= \frac{1}{2} \times$ hypotenuse [1/2]

\therefore In $\triangle PSR$,

$$SR = \frac{1}{2} \times PR$$

$\therefore 6 = \frac{1}{2} \times PR$

Ans. \therefore

$$PR = 12$$

[1/2] [1]

- (3) What is the distance between two parallel tangents of a circle having radius 4.5 cm?

Solution:

Radius of the circle = 4.5 cm

Distance between two parallel tangents of a circle = Diameter

$$\begin{aligned}
 & [1/2] \\
 &= 2r \\
 &= 2 \times 4.5 \\
 &= 9 \text{ cm}
 \end{aligned}$$

Ans. ∴ The distance between two parallel tangents is 9 cm.

[1/2] [1]

- (4) Find the co-ordinates of midpoint of the segment joining the points A(4, 6) and B(-2, 2).

Solution:

Let A(4, 6) = (x₁, y₁)

B(-2, 2) = (x₂, y₂)

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2} \quad [1/2]$$

$$\therefore x = \frac{4 - 2}{2}, \quad y = \frac{6 + 2}{2}$$

$$\therefore x = 1, \quad y = 4$$

Ans. ∴ (1, 4) is the midpoint of the given segment. [1/2] [1]

Q.2. (A) Complete the following activities and rewrite them.

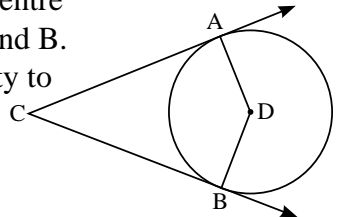
(Any two)

[4]

- (1) In the figure alongside, circle with centre D touches the sides of $\angle ACB$ at A and B.

If $\angle ACB = 52^\circ$, complete the activity to

find the measure of $\angle ADB$.



Activity:

In $\square ABCD$,

$$\angle CAD = \angle CBD = \boxed{90}^\circ \dots\dots\dots \text{Tangent theorem} \quad [1/2]$$

$$\therefore \angle ACB + \angle CAD + \angle CBD + \angle ADB = \boxed{360}^\circ \quad [1/2]$$

$$\therefore 52^\circ + 90^\circ + 90^\circ + \angle ADB = 360^\circ$$

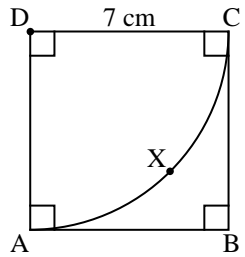
$$\therefore \angle ADB + \boxed{232}^\circ = 360^\circ \quad [1/2]$$

$$\therefore \angle ADB = 360^\circ - 232^\circ$$

Ans. ∴ $\angle ADB = \boxed{128}^\circ$

[1/2] [2]

- (2) In the figure alongside, side of square ABCD is 7 cm. With centre D and radius DA sector D–AXC is drawn. Complete the following activity to find the area of square ABCD and sector D–AXC.



Activity:

$$\begin{aligned} \text{Area of square} &= \boxed{(\text{side})^2} \dots\dots \text{formula} && [1/2] \\ &= (7)^2 \\ &= 49 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of sector (D – AXC)} &= \boxed{\frac{\theta}{360} \times \pi r^2} \dots\dots \text{formula} && [1/2] \\ &= \frac{\boxed{90}}{360} \times \frac{22}{7} \times \boxed{7^2} && [1/2 + 1/2] [2] \\ &= 38.5 \text{ cm}^2 \end{aligned}$$

- (3) Complete the following activity to prove $\cot \theta + \tan \theta = \text{cosec } \theta \times \sec \theta$.

Activity:

$$\text{L.H.S.} = \cot \theta + \tan \theta$$

$$= \frac{\boxed{\cos \theta}}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \quad [1/2]$$

$$= \frac{\boxed{\cos^2 \theta} + \boxed{\sin^2 \theta}}{\sin \theta \cdot \cos \theta} \quad [1/2 + 1/2]$$

$$= \frac{1}{\sin \theta \cdot \cos \theta} \quad (\because \sin^2 \theta + \cos^2 \theta = 1)$$

$$= \frac{1}{\sin \theta} \times \frac{1}{\cos \theta}$$

$$= \boxed{\text{cosec } \theta} \times \sec \theta \quad [1/2] [2]$$

$$\therefore = \text{R.H.S.}$$

$$\therefore \cot \theta + \tan \theta = \text{cosec } \theta \times \sec \theta$$

(B) Solve the following sub-questions. (Any four)

[8]

(1) If $\cos \theta = \frac{3}{5}$, then find $\sin \theta$.

Solution:

$$\cos \theta = \frac{3}{5} \quad \text{..... (given)}$$

$$\sin^2 \theta + \cos^2 \theta = 1 \quad [1/2]$$

$$\therefore \sin^2 \theta + \left(\frac{3}{5}\right)^2 = 1$$

$$\therefore \sin^2 \theta + \frac{9}{25} = 1$$

$$\therefore \sin^2 \theta = 1 - \frac{9}{25} \quad [1/2]$$

$$\therefore \sin^2 \theta = \frac{25 - 9}{25}$$

$$\therefore \sin^2 \theta = \frac{16}{25} \quad [1/2]$$

Ans. $\therefore \sin \theta = \frac{4}{5}$ (taking square root of both sides) [1/2] [2]

(2) Find the slope of line EF, where co-ordinates of E are $(-4, -2)$ and co-ordinates of F are $(6, 3)$.

Solution:

$$\text{Let } E(-4, -2) = (x_1, y_1)$$

$$F(6, 3) = (x_2, y_2)$$

$$\text{Slope of line EF} = \frac{y_2 - y_1}{x_2 - x_1} \quad [1/2]$$

$$= \frac{3 - (-2)}{6 - (-4)} \quad [1/2]$$

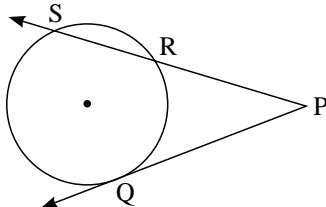
$$= \frac{3 + 2}{6 + 4}$$

$$= \frac{5}{10} \quad [1/2]$$

$$= \frac{1}{2}$$

Ans. \therefore Slope of line EF = $\frac{1}{2}$ [1/2] [2]

- (3) In the figure alongside, ray PQ touches the circle at point Q.
If $PQ = 12$, $PR = 8$,
find the length of seg PS.



Solution:

$$PQ^2 = PR \times PS \quad (\text{Tangent secant segments theorem}) \quad [1/2]$$

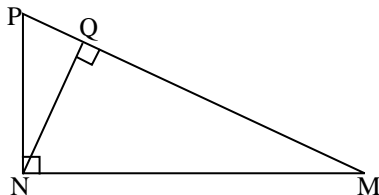
$$\therefore 12^2 = 8 \times PS \quad [1/2]$$

$$\therefore 144 = 8 \times PS \quad [1/2]$$

$$\therefore PS = \frac{144}{8}$$

Ans. $\therefore PS = 18$ [1/2] [2]

- (4) In the figure alongside,
 $\angle MNP = 90^\circ$,
seg $NQ \perp$ seg MP .
 $MQ = 9$, $QP = 4$. Find NQ .



Solution:

In $\triangle MNP$, $\angle N = 90^\circ$ and $NQ \perp PM$.

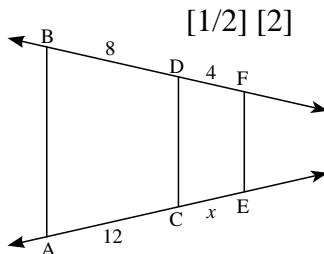
$$\therefore NQ^2 = QP \times MQ \quad (\text{Theorem of geometric mean}) \quad [1/2]$$

$$\therefore NQ^2 = 4 \times 9 \quad [1/2]$$

$$\therefore NQ^2 = 36 \quad [1/2]$$

Ans. $\therefore NQ = 6$ [1/2] [2]

- (5) In the figure alongside,
if $AB \parallel CD \parallel EF$, then find x
and AE by using the information
given in the figure.



Solution:

$AB \parallel CD \parallel EF$ and FB , EA are transversals.

$$\therefore \frac{BD}{DF} = \frac{AC}{CE} \quad (\text{Intercepts made by three parallel lines}) \quad [1/2]$$

$$\therefore \frac{8}{4} = \frac{12}{x}$$

$$\therefore 8x = 4 \times 12$$

$$\therefore x = \frac{4 \times 12}{8}$$

$$\therefore x = 6 \quad [1/2]$$

$$AE = AC + CE \quad (\text{A-C-E})$$

$$\therefore AE = 12 + x$$

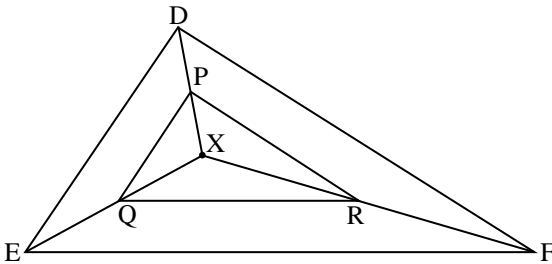
$$\therefore AE = 12 + 6$$

$$\therefore AE = 18 \quad [1/2] [2]$$

Ans. $x = 6$ and $AE = 18$

Q.3. (A) Complete the following activity and rewrite it.
(Any one) [3]

(1)



In the above figure, X is any point in the interior of triangle.

Point X is joined to vertices of triangle. $\text{seg } PQ \parallel \text{seg } DE$,
 $\text{seg } QR \parallel \text{seg } EF$. Complete the following activity to prove
 $\text{seg } PR \parallel \text{seg } DF$.

Activity:

In $\triangle XDE$, $PQ \parallel DE$ (given)

$$\therefore \frac{XP}{PD} = \frac{XQ}{QE} \quad \text{.....(I) Basic proportionality theorem} \quad [1/2 + 1/2]$$

In $\triangle XEF$, $QR \parallel EF$... (given)

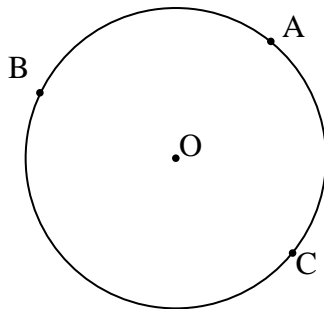
$$\therefore \frac{XQ}{QE} = \frac{XR}{RF} \quad \text{.....(II) } \boxed{\text{Basic proportionality theorem}} \quad [1/2 + 1/2]$$

$$\therefore \frac{XP}{PD} = \frac{XR}{RF} \quad \text{..... from (I) and (II) } \quad [1/2 + 1/2] [3]$$

$\therefore \text{seg } PR \parallel \text{seg } DF$ Converse of basic proportionality theorem

- (2) A, B, C are any points on the circle with centre O.

If $m(\text{arc BC}) = 110^\circ$ and $m(\text{arc AB}) = 125^\circ$, complete the following activity to find $m(\text{arc ABC})$, $m(\text{arc AC})$, $m(\text{arc ACB})$ and $m(\text{arc BAC})$.



Activity:

$$m(\text{arc ABC}) = m(\text{arc AB}) + \boxed{m(\text{arc BC})} \quad [1/2]$$

$$= \boxed{125^\circ} + 110^\circ \quad [1/2]$$

$$= 235^\circ$$

$$m(\text{arc AC}) = 360^\circ - m(\text{arc } \boxed{\text{ABC}}) \quad [1/2]$$

$$= 360^\circ - \boxed{235^\circ} \quad [1/2]$$

$$= 125^\circ$$

Similarly,

$$m(\text{arc ACB}) = 360^\circ - \boxed{125^\circ} \quad [1/2]$$

$$= 235^\circ$$

$$\text{and } m(\text{arc BAC}) = 360^\circ - \boxed{110^\circ} \quad [1/2] [3]$$

$$= 250^\circ$$

(B) Solve the following sub-questions. (Any two) [6]

- (1) The radius of a circle is 6 cm, the area of a sector of this circle is 15π sq. cm. Find the measure of the arc and the length of the arc corresponding to that sector.

Solution:

$$(i) \text{ Area of a sector} = \frac{l \times r}{2} \quad [1/2]$$

$$\therefore 15\pi = \frac{l \times 6}{2} \quad [1/2]$$

$$\therefore l = \frac{15\pi \times 2}{6}$$

$$\therefore l = 5\pi$$

Ans. Length of the arc = 5π cm [1/2]

$$(ii) \text{ Area of the sector} = \frac{\theta}{360} \pi r^2 \quad [1/2]$$

$$\therefore 15\pi = \frac{\theta}{360} \times \pi \times 6^2 \quad [1/2]$$

$$\therefore \theta = \frac{15\pi \times 360}{\pi \times 6 \times 6}$$

$$\therefore \theta = 15 \times 10$$

$$\therefore \theta = 150^\circ \quad [1/2] [3]$$

Ans. Measure of the arc is 150° .

(2) If A(3, 5) and B(7, 9), point Q divides seg AB in the ratio 2:3, find the co-ordinates of point Q.

Solution:

Let Q = (x, y) and

A(3, 5) = (x_1, y_1)

B(7, 9) = (x_2, y_2)

$m:n = 2:3$

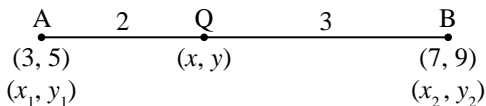
By section formula,

$$x = \frac{mx_2 + nx_1}{m+n};$$

$$= \frac{2 \times 7 + 3 \times 3}{2+3};$$

$$= \frac{14+9}{5};$$

$$= \frac{23}{5};$$



$$y = \frac{my_2 + ny_1}{m+n} \quad [1/2 + 1/2]$$

$$= \frac{2 \times 9 + 3 \times 5}{2+3} \quad [1/2 + 1/2]$$

$$= \frac{18+15}{5}$$

$$= \frac{33}{5} \quad [1/2 + 1/2] [3]$$

Ans. $\therefore Q\left(\frac{23}{5}, \frac{33}{5}\right)$

(3) Prove that:

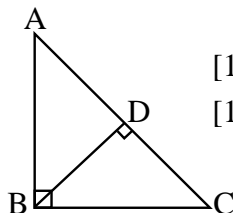
“In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of remaining two sides.”

Solution:

Given: In $\triangle ABC$, $\angle B = 90^\circ$ [1/2]

To prove: $AC^2 = AB^2 + BC^2$ [1/2]

Construction: Draw $BD \perp AC$



Proof:

$\triangle ABC \sim \triangle ADB$ (Similarity in right-angled triangles)

$$\therefore \frac{AB}{AD} = \frac{AC}{AB} \quad (\text{c.s.s.t})$$

$$\therefore AB^2 = AC \times AD \quad \dots\dots\dots(\text{I}) \quad [1/2]$$

$\triangle ABC \sim \triangle BDC$ (Similarity in right angled triangles)

$$\therefore \frac{BC}{DC} = \frac{AC}{BC}$$

$$\therefore BC^2 = AC \times DC \quad \dots\dots\dots(\text{II}) \quad [1/2]$$

Adding equation (I) and (II),

$$AB^2 + BC^2 = AC \times AD + AC \times DC \quad [1/2]$$

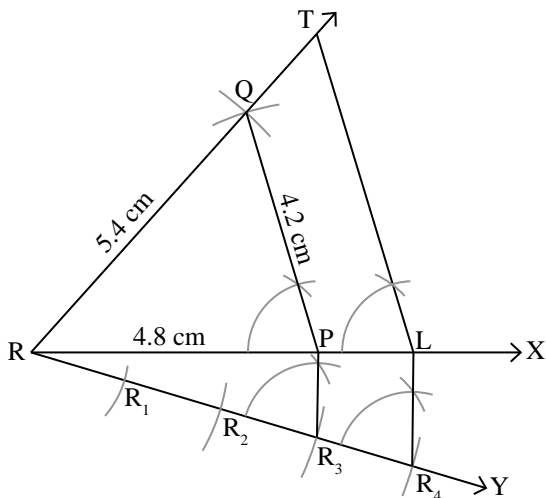
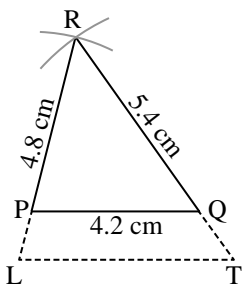
$$= AC(AD + DC)$$

$$= AC \times AC \quad \dots(\because A-D-C) \quad [1/2]$$

$$\therefore AB^2 + BC^2 = AC^2$$

(4) $\triangle PQR \sim \triangle LTR$. In $\triangle PQR$, $PQ=4.2$ cm, $QR=5.4$ cm, $PR=4.8$ cm.

Construct $\triangle PQR$ and $\triangle LTR$ such that $\frac{PQ}{LT} = \frac{3}{4}$.



- Draw $\triangle PQR$ of given measure [1]
- Draw acute angle at R [1/2]
- Mark points R_1, R_2, R_3, R_4 on Ray RY at equal distance from point R [1/2]

- Draw seg R_3Q and draw line R_4T parallel to it [1/2]
- Draw a line parallel to PQ through T [1/2] [3]

Q.4. Solve the following sub-questions. (Any two) [8]

- (1) A bucket is in the form of a frustum of a cone. It holds 28.490 litres of water. The radii of the top and the bottom are 28 cm and 21 cm respectively. Find the height of the bucket.

$$\left[\pi = \frac{22}{7} \right]$$

Solution:

$$\begin{aligned} \text{Volume of the frustum i.e. bucket (V)} &= 28.490 \text{ liters} \\ &= 28490 \text{ cm}^3 \quad [1/2] \end{aligned}$$

$$r_1 = 28 \text{ cm}, \quad r_2 = 21 \text{ cm}$$

$$\text{Volume of the bucket} = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 \times r_2) \quad [1/2]$$

$$\therefore 28490 = \frac{1}{3}\pi \times h(28^2 + 21^2 + 28 \times 21) \quad [1/2]$$

$$\therefore 28490 = \frac{1}{3} \times \frac{22}{7} \times h(784 + 441 + 588) \quad [1/2]$$

$$\therefore 28490 = \frac{1}{3} \times \frac{22}{7} \times h \times 1813 \quad [1/2]$$

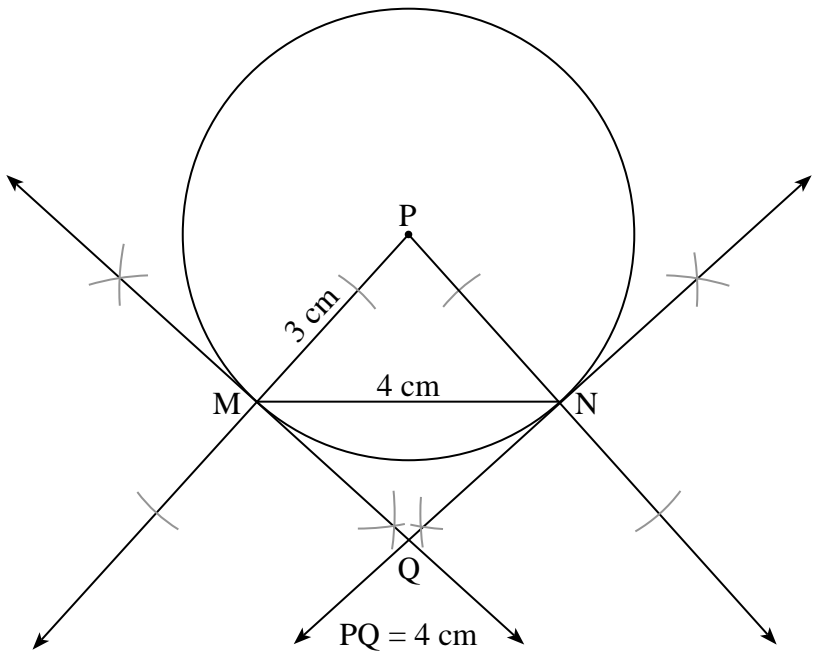
$$\therefore h = \frac{28490 \times 3 \times 7}{22 \times 1813} \quad [1/2]$$

$$\therefore h = 15 \quad [1/2]$$

Ans. \therefore The height of the bucket is 15 cm. [1/2] [4]

- (2) Draw a circle with centre P and radius 3 cm. Draw a chord MN of length 4 cm. Draw tangents to the circle through points M and N which intersect in point Q . Measure the length of seg PQ .

Solution:



- To draw a circle with centre P and radius 3 cm [1]
- To chord MN of length 4 cm [1]
- To draw tangents at M and N [1]
- To measure the length PQ [1]

(3) In ΔPQR , bisectors of $\angle Q$ and $\angle R$ intersect in point X. Line PX intersects side QR in point Y, then prove that:

$$\frac{PQ + PR}{QR} = \frac{PX}{XY}$$

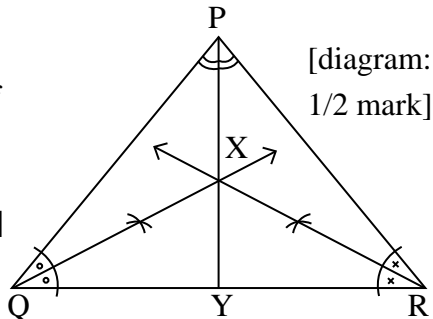
Solution:

Given: In ΔPQR , rays QX and RX are angle bisectors of $\angle Q$ and $\angle R$ respectively.

[diagram:
1/2 mark]

To prove: $\frac{PQ + PR}{QR} = \frac{PX}{XY}$ [1/2]

Proof: In ΔPQY , ray QX is a angle bisector of $\angle PQY$.



$$\therefore \frac{PX}{XY} = \frac{PQ}{QY} \dots\dots\dots\text{(I) (Angle bisector theorem) [1]}$$

In $\triangle PRY$, ray RX is an angle bisector of $\angle PRY$.

$$\therefore \frac{PX}{XY} = \frac{PR}{RY} \dots\dots\text{(II) (Angle bisector theorem) [1/2]}$$

$$\therefore \frac{PX}{XY} = \frac{PQ}{QY} = \frac{PR}{RY} \dots\dots\text{[From (I) \& (II)] [1/2]}$$

$$\therefore \frac{PX}{XY} = \frac{PQ + PR}{QY + RY} \dots\dots\text{(Theorem on equal ratio) [1/2]}$$

$$\therefore \frac{PX}{XY} = \frac{PQ + PR}{QR} \dots\dots\text{(Q-Y-C) [1/2] [4]}$$

Q.5. Solve the following sub-questions. (Any one) [3]

(1) From top of the building, Ramesh is looking at a bicycle parked at some distance away from the building on the road.

If

$AB \rightarrow$ Height of building is 40 m

$C \rightarrow$ Position of bicycle

$A \rightarrow$ Position of Ramesh on top of the building

$\angle MAC$ is the angle of depression and $m\angle MAC = 30^\circ$, then:

- (a) Draw a figure with the given information.
- (b) Find the distance between building and the bicycle ($\sqrt{3} = 1.73$).

Solution:

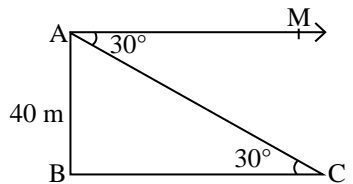
$$\angle MAC = 30^\circ$$

$$\therefore \angle ACB = 30^\circ \quad \text{(alternate angles)}$$

$\triangle ABC$ is a right-angled triangle.

$$\tan 30^\circ = \frac{AB}{BC}$$

$$\therefore \frac{1}{\sqrt{3}} = \frac{40}{BC}$$



[diagram:
1 mark]

[1/2]

$$\therefore BC = 40\sqrt{3} \quad [1/2]$$

$$\therefore BC = 40 \times 1.73 \quad [1/2]$$

$$\therefore BC = 69.20 \text{ m}$$

Ans. \therefore The distance between the building and the bicycle is 69.20 m. [1/2] [3]

(2) $\square ABCD$ is a cyclic quadrilateral where side $AB \cong$ side BC , $\angle ADC = 110^\circ$, AC is the diagonal, then:

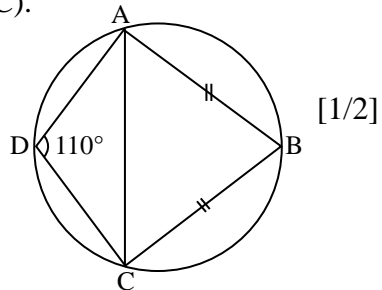
(a) Draw the figure using given information.

(b) Find measure of $\angle ABC$.

(c) Find measure of $\angle BAC$.

(d) Find measure of (arc ABC).

(a)



(b) In cyclic quadrilateral $ABCD$,

$$m\angle ADC + m\angle ABC = 180^\circ$$

(opposite angles of cyclic quadrilateral)

[1/2]

$$\therefore 110 + m\angle ABC = 180^\circ$$

$$\therefore m\angle ABC = 180^\circ - 110^\circ$$

$$\therefore m\angle ABC = 70^\circ \quad [1/2]$$

(c) $\triangle ABC$ is an isosceles triangle. ($\because AB = BC$)

$$\therefore m\angle ACB = m\angle BAC \quad (\text{Base angles of an isosceles triangle})$$

[1/2]

$$\therefore m\angle BAC + m\angle ACB + m\angle ABC = 180^\circ$$

$$\therefore m\angle BAC + m\angle BAC + 70^\circ = 180^\circ$$

$$\therefore 2\angle BAC = 180^\circ - 70^\circ$$

$$\therefore \angle BAC = \frac{110^\circ}{2}$$

$$\therefore \angle BAC = 55^\circ \quad [1/2]$$

(d) $m\angle ADC = \frac{1}{2} m(\text{arc } ABC)$ (Inscribed angle theorem)

$$\therefore 110^\circ = \frac{1}{2} m(\text{arc } ABC)$$

$$\therefore m(\text{arc } ABC) = 2 \times 110^\circ$$

$$\therefore m(\text{arc } ABC) = 220^\circ \quad [1/2] [3]$$

★★★