SOLUTION

Q.1. (A) For every sub-question four alternative answers are given. Choose the correct answer and write its alphabet. [4]

(1) For an A.P., $a = 3.5$, $d = 0$, then $t_n = ...$

(a) 0 (b) 3.5 (c) 103.5 (d) 104.5 [1]

(2) Find the value of the determinant $\begin{vmatrix} 5 & 3 \\ -7 & -4 \end{vmatrix}$:

(a) –1 (b) –41 (c) 41 (d) 1 [1]

(3) Which of the following quadratic equations has roots 3 and 5?

(a) $x^2 - 15x + 8 = 0$ (b) $x^2 + 8x - 15 = 0$

(c) $x^2 + 3x + 5 = 0$ (d) $x^2 - 8x + 15 = 0$ [1]

(4) There are 40 cards in a bag. Each card bears a number from 1 to 40. One card is drawn at random. What is the probability that the card bears a number which is a multiple of 5?

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{1}{3}$ [1]

Ans. (1) - (b), (2) - (d), (3) - (d), (4) - (a) [4]

Q.1. (B) Solve the following sub-questions. [4]

(1) The sum of the father's age and twice the age of his son is 70. Use the given information to form a linear equation in two variables.

Solution:

Let the father's age be 'x' years and that of son be 'y' years.

According to the given information,

Ans. $\boxed{x + 2y = 70}$ [1]

(2) A die is thrown. Write the sample space.

Solution: A die is thrown.

Ans. Sample space $\boxed{S = \{1, 2, 3, 4, 5, 6\}}$ [1]

(3) Find the roots of the quadratic equation $(x + 5)(x - 4) = 0$.

Solution:

$(x + 5)(x - 4) = 0$ .... (given)

$\therefore \quad x + 5 = 0 \quad$ or $\quad x - 4 = 0$ ...[½]

$\therefore \quad \boxed{x = -5 \quad \text{or} \quad x = 4}$ ...[½] [1]

Ans. The roots of the given quadratic equation are –5, 4.

(4) Find the first term and the common difference for the A.P.

127, 135, 143, 151, ...

Solution:

First term $= t_1 = a = \boxed{127}$ ...[½]

$t_2 = 135, t_3 = 143, t_4 = 151$

Common difference $= d = t_2 - t_1$

$= 135 - 127$

$\therefore \quad \boxed{d = 8}$ ...[½] [1]

Similarly, $d = t_3 - t_2 = t_4 - t_3 = 8$

Ans. The first term is 127 and the common difference is 8.

Q.2. (A) Complete the following activities and rewrite them. (Any *two*) **[4]**

(1) Complete the following activity to find the 27th term of the A.P.:

9, 4, –1, –6, –11, ...

Activity:

Here $a = 9$, $d = \boxed{}$, $n = 27$

$t_n = \boxed{} + (n - 1)d$ .... (formula)

$\therefore \quad t_{27} = 9 + (\boxed{} - 1)(-5)$

$\therefore \quad t_{27} = \boxed{}$

Solution:

$a = 9$, $d = \boxed{-5}$, $n = 27$ ...[½]

$t_n = \boxed{a} + (n - 1)d$ .... (formula) ...[½]

$\therefore \quad t_{27} = 9 + (\boxed{27} - 1)(-5)$ ...[½]

$\therefore \quad t_{27} = \boxed{-121}$ ...[½] [2]

(2) One die is rolled. Complete the following activity, to find the probability that the number on the upper face is prime.

Activity:

'S' is the sample space.

$\therefore \quad S = \{\boxed{}\}$

$\therefore n(S) = 6$

Event A: Getting prime number on the upper face.

$\therefore \quad A = \{\boxed{}\}$

$\therefore n(A) = 3$

$P(A) = \dfrac{n(A)}{\boxed{}}$ .... (formula)

$\therefore P(A) = \boxed{}$

Solution:

'S' is the sample space.

$\therefore \quad S = \{\boxed{1, 2, 3, 4, 5, 6}\}$ ...[½]

$\therefore \quad n(S) = 6$

Event A: Getting prime number on the upper face.

$\therefore \quad A = \{\boxed{2, 3, 5}\}$ ...[½]

$\therefore \quad n(A) = 3$

$P(A) = \dfrac{n(A)}{\boxed{n(S)}}$ .... (formula) ...[½]

$\therefore \boxed{\boldsymbol{P(A) = \dfrac{1}{2}}}$ ...[½] [2]

(3) Complete the following activity to find the value of *x*.

Activity:

$$\begin{array}{rl} & 3x - y = 2 \\ + & 2x + y = 8 \\ \hline & \boxed{}x = \boxed{} \end{array}$$

$\therefore \quad x = \dfrac{\boxed{}}{5}$

$\therefore \quad x = \boxed{}$

Solution :

$$\begin{array}{r} 3x - y = 2 \\ + \quad 2x + y = 8 \\ \hline \boxed{5}\,x = \boxed{10} \end{array}$$ ...[½] + [½]

$\therefore \quad x = \dfrac{\boxed{10}}{5}$ ...[½]

$\therefore \quad x = \boxed{\mathbf{2}}$ ...[½] [2]

Q.2. (B) Solve the following sub-questions. (Any *four*) [8]

(1) For solving the following simultaneous equations, find the values of $(x + y)$ and $(x - y)$.

$\mathbf{15x + 17y = 21}$

$\mathbf{17x + 15y = 11}$

Solution :

Adding the two given equations,

$$\begin{array}{r} 15x + 17y = 21 \\ + \quad 17x + 15y = 11 \\ \hline 32x + 32y = 32 \end{array}$$ ...[½]

$\therefore \quad \boxed{\mathbf{x + y = 1}}$ ...[½]

Subtracting the two given equations,

$$\begin{array}{r} 15x + 17y = 21 \\ - \quad 17x + 15y = 11 \\ - \quad\ - \quad\ - \\ \hline -2x + 2y = 10 \end{array}$$ ...[½]

$\therefore \quad -2(x - y) = 10$

$\therefore \quad x - y = \dfrac{-10}{2}$

$\therefore \quad \boxed{\mathbf{x - y = -5}}$ ...[½] [2]

(2) Find the value of the discriminant of the quadratic equation.

$\mathbf{2y^2 - y + 2 = 0}$

Solution :

$2y^2 - y + 2 = 0$ ....(given)

$\therefore$ Comparing the equation with $ax^2 + bx + c = 0$,

$a = 2, \quad b = -1, \quad c = 2$ ...[½]

Discriminant $= \Delta = b^2 - 4ac$ ...[½]

$\therefore$ $= (-1)^2 - 4(2)(2)$ ...[½]

$\therefore$ $= 1 - 16$

Ans. $\boxed{\Delta = \mathbf{-15}}$ ...[½] [2]

(3) Find the sum of the first 21 even natural numbers.

Solution:

Even natural numbers are 2, 4, 6, 8, ...

Here, $a = t_1 = 2,\ t_2 = 4,\ t_3 = 6,\ t_4 = 8$ ...[½]

$d = t_2 - t_1 = 4 - 2 = 2$

Also, $d = t_3 - t_2 = 6 - 4 = 2$

$d = t_4 - t_3 = 8 - 6 = 2$

$\therefore$ Even natural numbers are in A.P. with $d = 2$.

Also $n = 21$

To find S_{21} :

$S_n = \frac{n}{2}[2a + (n - 1)d]$ ....(formula) ...[½]

$= \frac{21}{2}[2(2) + (21 - 1)2]$ ...[½]

$= \frac{21}{2}[4 + 20(2)]$

$= \frac{21}{2}[44]$

$= 21 \times 22$

$\therefore$ $S_{21} = \boxed{\mathbf{462}}$ ...[½] [2]

Ans. The sum of the first 21 even natural numbers is 462.

(4) Two coins are tossed simultaneously. Find the probability of the event of getting 'no head'.

Solution:

Two coins are tossed.

$\therefore$ S = {HH, HT, TH, TT} ...[½]

$\therefore$ $n(S) = 4$

Let event A: getting no head

$\therefore \quad A = \{TT\}$ ...[½]

$\therefore \quad n(A) = 1$

$P(A) = \dfrac{n(A)}{n(S)}$ ....(formula) ...[½]

$\therefore \quad \boxed{\boldsymbol{P(A) = \frac{1}{4}}}$ ...[½] [2]

Ans. The probability of getting 'no head' is $\frac{1}{4}$.

(5) Find D_x and D_y for the following simultaneous equations.

$$\boldsymbol{x + 2y = -1, \quad 2x - 3y = 12}$$

Solution:

$$\left.\begin{aligned} x + 2y &= -1 \\ 2x - 3y &= 12 \end{aligned}\right\} \text{(given)}$$

$\therefore \quad D_x = \begin{vmatrix} -1 & 2 \\ 12 & -3 \end{vmatrix}$ ...[½]

$= [(-1) \times (-3)] - [2 \times 12]$

$= 3 - 24$

$\therefore \quad \boxed{D_x = -21}$ ...[½]

$D_y = \begin{vmatrix} 1 & -1 \\ 2 & 12 \end{vmatrix}$ ...[½]

$= [12 \times 1] - [2 \times (-1)]$

$= 12 + 2$

$\therefore \quad \boxed{D_y = 14}$ ...[½] [2]

Q.3. (A) Complete the following activity and rewrite it. (Any *one*) **[3]**

(1) From three men and two women, an environment committee of two persons is to be formed. To find the probabilities of the given events, complete the following activities.

Event A: There must be at least one woman member.

Event B: Committee of one man and one woman to be formed.

Activity:

Let M_1, M_2, M_3 be three men, and W_1, W_2 be two women. Out of these men and women, an environment committee of two persons is to be formed.

$$S = \{M_1M_2, M_1M_3, M_2M_3, M_1W_1, M_1W_2, M_2W_1, M_2W_2, M_3W_1, M_3W_2, \boxed{}\}$$

$\therefore\ n(S) = 10$

Event A: There must be at least one woman member.

$$\therefore \quad A = \{M_1W_1, M_1W_2, \boxed{}, M_2W_2, M_3W_1, M_3W_2, W_1W_2\}$$

$$\therefore \quad n(A) = \boxed{}$$

$$P(A) = \frac{n(A)}{n(S)} \quad \text{........... (formula)}$$

$$\therefore \quad P(A) = \frac{\boxed{}}{10}$$

Event B: Committee of one man and one woman to be formed.

$$\therefore \quad B = \{M_1W_1, M_1W_2, M_2W_1, \boxed{}, M_3W_1, M_3W_2\}$$

$$\therefore \quad n(B) = 6$$

$$P(B) = \frac{n(B)}{n(S)} \quad \text{........... (formula)}$$

$$\therefore \quad P(B) = \frac{6}{10}$$

$$\therefore \quad P(B) = \frac{3}{\boxed{}}$$

Solution:

Let M_1, M_2, M_3 be three men and W_1, W_2 be two women. Out of these men and women, an environment committee of the two persons is to be formed.

$S = \{M_1M_2, M_1M_3, M_2M_3, M_1W_1, M_1W_2, M_2W_1, M_2W_2, M_3W_1, M_3W_2, \boxed{W_1W_2}\}$ ...[½]

$$\therefore \quad n(S) = 10$$

Event A: There must be at least one woman member.

$\therefore \quad A = \{M_1W_1, M_1W_2, \boxed{M_2W_1}, M_2W_2, M_3W_1, M_3W_2, W_1W_2\}$ ...[½]

$\therefore \quad n(\text{A}) = \boxed{7}$ ...[½]

$$P(\text{A}) = \frac{n(\text{A})}{n(\text{S})} \text{ .... (formula)}$$

$\therefore \quad P(\text{A}) = \frac{\boxed{7}}{10}$ ...[½]

Event B: Committee of one man and one woman to be formed.

$\therefore \quad \text{B} = \{\text{M}_1\text{W}_1, \text{M}_1\text{W}_2, \text{M}_2\text{W}_1, \boxed{\text{M}_2\text{W}_2}, \text{M}_3\text{W}_1, \text{M}_3\text{W}_2\}$ ...[½]

$\therefore \quad n(\text{B}) = 6$

$$P(\text{B}) = \frac{n(\text{B})}{n(\text{S})} \text{ .... (formula)}$$

$\therefore \quad P(\text{B}) = \frac{6}{10}$

$\therefore \quad P(\text{B}) = \frac{3}{\boxed{5}}$ ...[½] [3]

(2) Complete the following activity to find the roots of the quadratic equation by the formula method.

$$\mathbf{25x^2 + 30x + 9 = 0}$$

Activity:

$$25x^2 + 30x + 9 = 0$$

Comparing the equation with $ax^2 + bx + c = 0$,

we get $a = 25$, $b = \boxed{}$, $c = 9$

$\therefore \quad b^2 - 4ac = (30)^2 - 4 \times 25 \times 9$

$= 900 - 900$

$= \boxed{}$

$\therefore \quad x = \frac{\boxed{} \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore \quad x = \frac{-\boxed{} \pm \sqrt{0}}{2 \times 25}$

$\therefore \quad x = \frac{-30 + 0}{50}$ *or* $\therefore \quad x = \frac{\boxed{} - 0}{50}$

$\therefore \quad x = -\frac{30}{50} \qquad or \qquad \therefore \; x = -\frac{30}{50}$

$\therefore \quad x = -\frac{\boxed{}}{5} \qquad or \qquad \therefore \; x = -\frac{3}{5}$

Solution:

$$25x^2 + 30x + 9 = 0$$

Comparing the equation with $ax^2 + bx + c = 0$,

we get $a = 25$, $b = \boxed{30}$, $c = 9$ ...[½]

$\therefore \quad b^2 - 4ac = (30)^2 - 4 \times 25 \times 9$

$= 900 - 900$

$= \boxed{0}$ ...[½]

$\therefore \quad x = \frac{\boxed{-b} \pm \sqrt{b^2 - 4ac}}{2a}$ ...[½]

$\therefore \quad x = \frac{-\boxed{30} \pm \sqrt{0}}{2 \times 25}$ ...[½]

$\therefore \; x = \frac{-30 + 0}{50} \qquad or \qquad \therefore \quad x = \frac{\boxed{-30} - 0}{50}$ ...[½]

$\therefore \; x = -\frac{30}{50} \qquad or \qquad \therefore \quad x = -\frac{30}{50}$

$\therefore \; x = -\frac{\boxed{3}}{5} \qquad or \qquad \therefore \quad x = -\frac{3}{5}$ ...[½] [3]

Q.3. (B) Solve the following sub-questions. (Any *two*) **[6]**

(1) Solve the given equation by factorisation:

$$\mathbf{5m^2 = 22m + 15}$$

Solution:

$5m^2 = 22m + 15$ .... (given)

$\therefore \quad 5m^2 - 22m - 15 = 0$ ...[½]

$\therefore \quad 5m^2 - 25m + 3m - 15 = 0$ ...[½]

$\therefore \quad 5m(m - 5) + 3(m - 5) = 0$ ...[½]

$\therefore \quad (m - 5)(5m + 3) = 0$

$\therefore \quad m - 5 = 0 \quad or \quad 5m + 3 = 0$ ...[½]

$\therefore \quad \boxed{m = 5 \quad or \quad m = -\frac{3}{5}}$ ...[½] + [½] [3]

Ans. $\boxed{\boldsymbol{m = 5, -\frac{3}{5}}}$

(2) Solve the following equations.

$$\mathbf{3x - 2y = \frac{5}{2}, \quad \frac{1}{3}x + 3y = -\frac{4}{3}}$$

Solution:

$$3x - 2y = \frac{5}{2}, \quad \frac{1}{3}x + 3y = -\frac{4}{3}$$

$$D = \begin{vmatrix} 3 & -2 \\ 1/3 & 3 \end{vmatrix}$$

$$= 3 \times 3 - \frac{1}{3} \times (-2) = 9 + \frac{2}{3} = \frac{27+2}{3} = \frac{29}{3}$$ ...[½]

$$D_x = \begin{vmatrix} 5/2 & -2 \\ -4/3 & 3 \end{vmatrix}$$

$$= \frac{5}{2} \times 3 - (-2) \times \frac{-4}{3} = \frac{15}{2} - \frac{8}{3} = \frac{45-16}{6} = \frac{29}{6}$$ ...[½]

$$D_y = \begin{vmatrix} 3 & 5/2 \\ 1/3 & -4/3 \end{vmatrix}$$

$$= 3 \times \frac{-4}{3} - \frac{5}{2} \times \frac{1}{3} = -4 - \frac{5}{6} = \frac{-24-5}{6} = -\frac{29}{6}$$ ...[½]

By Cramer's rule, ...[½]

$$x = \frac{D_x}{D} = \frac{29}{6} \times \frac{3}{29} = \boxed{\frac{1}{2}}$$ ...[½]

and $y = \frac{D_y}{D} = \frac{-29}{6} \times \frac{3}{29} = \boxed{\frac{-1}{2}}$ ...[½] [3]

Ans. $\boxed{\boldsymbol{x = \frac{1}{2}}}$ **and** $\boxed{\boldsymbol{y = \frac{-1}{2}}}$

(3) The length and breadth of a rectangular garden are 77 meters and 50 meters, respectively. There is a circular lake in the garden, having a diameter of 14m. Due to wind, a towel from a terrace on a nearby building fell into the garden. Then find the probability of the event that it fell in the lake.

Solution:

Area of a rectangular garden = length × breadth ...[½]

$= 77 \times 50$ .... (given)

$= \boxed{3850\ m^2}$ ...[½]

Area of a circular lake $= \pi \times (\text{radius})^2$ ...[½]

$$= \pi \times \left(\frac{\text{diameter}}{2}\right)^2$$

$$= \pi \left(\frac{14}{2}\right)^2 \text{ .... } (d = 14, \text{ given})$$

$$= \frac{22}{7} \times \frac{14}{2} \times \frac{14}{2}$$

$= \boxed{154\ m^2}$ ...[½]

The probability of the event that the towel fell in the lake

$$= \frac{\text{area of the lake}}{\text{area of the garden}}$$ ...[½]

$$= \frac{154}{3850}$$

$$= \boxed{\frac{1}{25}}$$ ...[½] [3]

Ans. Probability is $\boxed{\mathbf{\frac{1}{25}}}$

(4) A two-digit number and the number with digits interchanged add up to 143. In the given number, the digit in the units place is 3 more than the digit in the tens place. Find the original number.

Solution:

Let 'x' be the digit in the unit's place and 'y' be the digit in the ten's place. Then the required number will be $(x + 10y)$ ...[½]

According to the given conditions,

$$(x + 10y) + (y + 10x) = 143$$

$$\therefore \quad 11x + 11y = 143$$

$$\therefore \quad x + y = 13 \quad \text{.....................(1)} \qquad \text{...[½]}$$

$$\text{and} \quad x = 3 + y$$

$$\therefore \quad x - y = 3 \quad \text{.....................(2)} \qquad \text{...[½]}$$

Adding equations (1) and (2),

$$x + y + x - y = 13 + 3$$

$$\therefore \quad 2x = 16$$

$$\therefore \quad \boxed{x = 8} \qquad \text{...[½]}$$

Substituting $x = 8$ in equation (2),

$$8 - y = 3$$

$$\therefore \quad -y = 3 - 8$$

$$\therefore \quad \boxed{y = 5} \qquad \text{...[½]}$$

$$\text{Original number} = x + 10y$$

$$= 8 + 10 \times 5$$

$$= \boxed{58} \qquad \text{...[½] [3]}$$

Ans. The required number is 58.

Q.4. Solve the following sub-questions. (Any *two*) **[8]**

(1) Solve the following simultaneous equations graphically.

$$\boldsymbol{x + y = 4}$$

$$\boldsymbol{3x - 2y = 7}$$

Solution:

(a) $x + y = 4$

$\boldsymbol{x}$	0	1	2	3
$\boldsymbol{y}$	4	3	2	1
$\boldsymbol{(x, y)}$	(0, 4)	(1, 3)	(2, 2)	(3, 1)

...[½]

$x + y = 4$

(i) Put $\boxed{x = 0}$

$\therefore \quad 0 + y = 4$

$\therefore \quad \boxed{y = 4}$

(ii) Put $\boxed{x = 1}$

$\therefore \quad 1 + y = 4$

$\therefore \quad \boxed{y = 3}$

(iii) Put $\boxed{x = 2}$

$\therefore \quad 2 + y = 4$

$\therefore \quad \boxed{y = 2}$

(iv) Put $\boxed{x = 3}$

$\therefore \quad 3 + y = 4$

$\therefore \quad \boxed{y = 1}$

(b) $3x - 2y = 7$

x	0	1	2	3
y	–3.5	–2	–0.5	1
(x, y)	(0, –3.5)	(1, –2)	(2, –0.5)	(3, 1)

...[½]

$3x - 2y = 7$

(i) Put $\boxed{x = 0}$

$\therefore \quad 3(0) - 2y = 7$

$\therefore \quad y = \dfrac{-7}{2}$

$\therefore \quad \boxed{y = -3.5}$

(ii) Put $\boxed{x = 1}$

$\therefore \quad 3(1) - 2y = 7$

$\therefore \quad y = \dfrac{4}{-2}$

$\therefore \quad \boxed{y = -2}$

(iii) Put $\boxed{x = 2}$

$\therefore \quad 3(2) - 2y = 7$

$\therefore \quad y = \dfrac{-1}{2}$

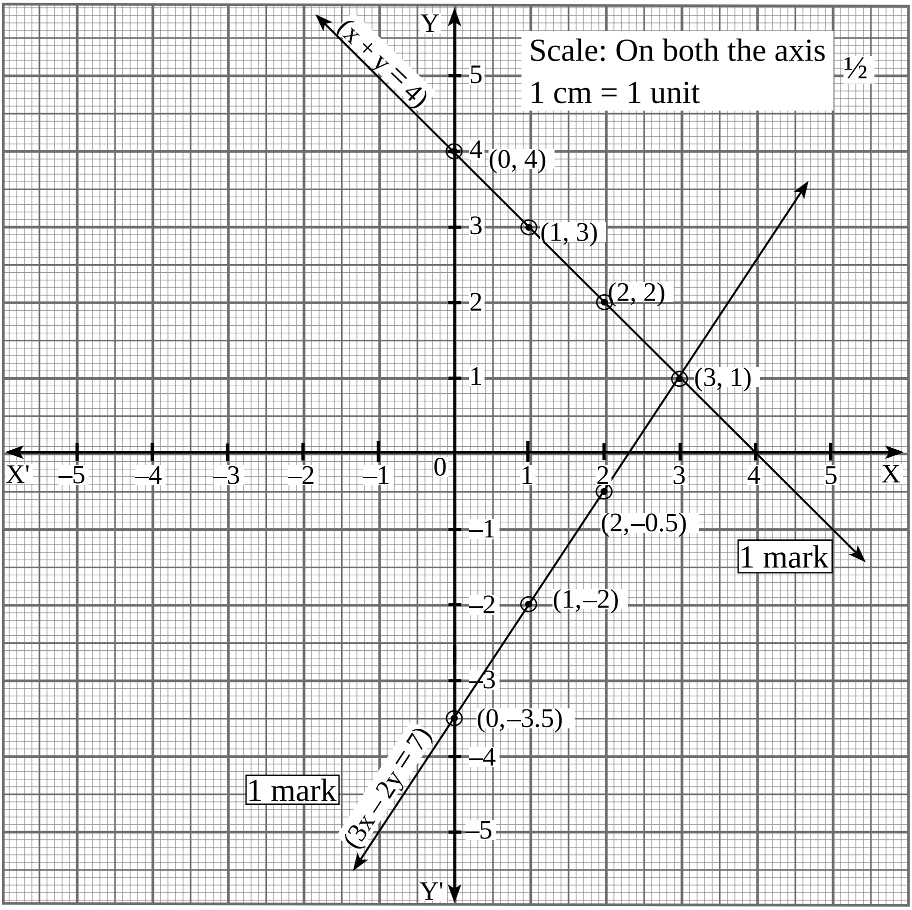
$\therefore \quad \boxed{y = -0.5}$

(iv) Put $\boxed{x = 3}$

$\therefore \quad 3(3) - 2y = 7$

$\therefore \quad y = \dfrac{-2}{-2}$

$\therefore \quad \boxed{y = 1}$



The two lines intersect each other at point (3, 1).

$\therefore$ The solution is (3, 1).

Ans. $\boxed{x = 3}$ $\boxed{y = 1}$ ...[½] [4]

(2) A train travels 240 km with uniform speed. If the speed of the train is increased by 12 km/hr, it takes one hour less to cover the same distance. Find the initial speed of the train.

Solution:

Let 'x' km/hr be the initial speed of the train.

$\therefore$ Time taken $t_1 = \dfrac{\text{distance}}{\text{speed}} = \dfrac{240}{x}$ hr ...[½]

When the speed is increased by 12 km/hr, the time taken is $t_2 = \dfrac{240}{x+12}$ hr ...[½]

According to the given condition,

$$t_2 = t_1 - 1$$

$\therefore$ $$t_1 - t_2 = 1$$

$\therefore$ $$\frac{240}{x} - \frac{240}{x+12} = 1$$ ...[½]

$\therefore$ $$\frac{240x + 2880 - 240x}{x(x+12)} = 1$$

$\therefore$ $$x^2 + 12x - 2880 = 0$$ ...[½]

$\therefore$ Adding 36 to both sides,

$$x^2 + 12x + 36 = 2880 + 36$$ ...[½]

$\therefore$ $$(x+6)^2 = 2916$$ ...[½]

Taking Square root,

$$x + 6 = \pm 54$$

$\therefore$ $x = -54 - 6$ *or* $x = 54 - 6$

$\therefore$ $x = -60$ *or* $\boxed{\mathbf{x = 48\ km/hr}}$ ...[½]

Discarding $x = -60$ as speed can't be negative. ...[½] [4]

Ans. The initial speed of the train is 48 km/hr.

(3) If the sum of the first p terms of an A.P. is equal to the sum of the first q terms, then show that the sum of its first $(p + q)$ terms is zero $(p \neq q)$.

Solution:

The sum of the first n terms of an A.P. is

$$S_n = \frac{n}{2}[2a + (n-1)d],$$ ...[½]

where a is the first term and d is the common difference.

$S_p = S_q$ ....(given)

$\therefore$ $$\frac{p}{2}[2a + (p-1)d] = \frac{q}{2}[2a + (q-1)d]$$ ...[½]

$\therefore$ $$p[2a + (p-1)d] = q[2a + (q-1)d]$$

$\therefore \quad 2ap + p(p-1)d = 2aq + q(q-1)d$

$\therefore \quad 2ap + p^2d - pd = 2aq + q^2d - qd$ ...[½]

$\therefore \quad 2ap - 2aq = q^2d - qd - p^2d + pd$

$\therefore \quad 2a(p-q) = d(q^2 - p^2) + d(p-q)$

$\therefore \quad 2a(p-q) = d[(q^2 - p^2) + (p-q)]$ ...[½]

$\therefore \quad 2a(p-q) = d[(q+p)(q-p) + (p-q)]$

$\therefore \quad 2a(p-q) = d[(q+p) \times (-1)(p-q) + (p-q)]$

$\therefore \quad 2a(p-q) = d[-(q+p)(p-q) + (p-q)]$

$\therefore \quad 2a(p-q) = d(p-q)[-(q+p) + 1]$

$\therefore \quad 2a(p-q) = d(p-q)(1-q-p)$ ...[½]

$p \neq q$

$\therefore \quad p - q \neq 0$

Dividing by $(p-q)$ we get,

$2a = d(1-q-p)$ ....(1) ...[½]

$S_{p+q} = \left(\dfrac{p+q}{2}\right)[2a + (p+q-1)d]$ ...[½]

$= \left(\dfrac{p+q}{2}\right)[d(1-q-p) + (p+q-1)d]$ ....[using (1)]

$= \left(\dfrac{p+q}{2}\right)[1-q-p+p+q-1]d$

$\therefore \quad S_{p+q} = 0$ ...[½] [4]

Ans. Hence, proved that the sum of the first $(p+q)$ terms is zero.

Q.5. Solve the following sub-questions. (Any *one*) **[3]**

(1) The measures of the angles of a quadrilateral are in A.P. The measure of the largest angle is twice the smallest. Find the measures of all angles of the quadrilateral.
(Assume measures of angles as a, $a+d$, $a+2d$, $a+3d$, where $a < a+d < a+2d < a+3d$.)

Solution:

The measures of the angles of a quadrilateral are in A.P. ...(given)

$\therefore$ Let 'a' be the the measure of the smallest angle and 'd' be the common difference. Then, the angles are a, $a + d$, $a + 2d$ and $a + 3d$.

$\therefore$ $a + (a + d) + (a + 2d) + (a + 3d) = 360°$ ...(sum of the angles of a quadrilateral)

$\therefore$ $4a + 6d = 360°$

$\therefore$ $2a + 3d = 180°$ ....(1) ...[½]

According to the given condition,

$(a + 3d) = 2a$

$\therefore$ $-a + 3d = 0$

$\therefore$ $-2a + 6d = 0$ ....(2) ...[½]

Adding equation (1) and (2),

$9d = 180°$

$\therefore$ $\boxed{d = 20°}$ ...[½]

$\therefore$ $\boxed{a = 60°}$ ....[using (1)] ...[½]

$\therefore$ The angles are

$a = \boxed{60°}$

$a + d = 60° + 20° = \boxed{80°}$

$a + 2d = 60° + 2(20)° = \boxed{100°}$

$a + 3d = 60° + 3(20)° = \boxed{120°}$ [1] [3]

Ans. The angles are 60°, 80°, 100° and 120°.

(2) The product of two numbers is 352 and their mean is 19. Find the numbers.

Solution:

Let the two numbers be x and y.

According to the given conditions,

$xy = 352 \Rightarrow y = \dfrac{352}{x}$ ....(1) ...[½]

and $\dfrac{x + y}{2} = 19 \Rightarrow x + y = 38$ ....(2)

...[½]

by (1) and (2), $x + \dfrac{352}{x} = 38$

$$x^2 - 38x + 352 = 0 \qquad ...[½]$$

$$x^2 - 22x - 16x + 352 = 0$$

$$x(x - 22) - 16(x - 22) = 0$$

$$(x - 22)(x - 16) = 0 \qquad ...[½]$$

$$x - 22 = 0 \quad \text{or} \quad x - 16 = 0$$

$$\boxed{x = 22} \quad \text{or} \quad \boxed{x = 16} \qquad ...[½]$$

For $x = 22$, $y = \dfrac{352}{22} = \boxed{16}$ ....[using (1)]

For $x = 16$, $y = \dfrac{352}{16} = \boxed{22}$ ....[using (1)]

Ans. The two numbers are $\boxed{\textbf{16 and 22}}$ ...[½] [3]

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